PROCEEDINGS

American Society of Civil Engineers

OCTOBER

1946

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

VOLUME 72

NO. 8

PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 72 OCTOBER, 1946 No. 8

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except July and August, at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-Ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society are given.

Entered as Second-Class Matter, September 23, 1937, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy

Printed in the United States of America

Current Papers and Discussions

	Published	Discussion closes
Symposium: Cavitation in Hydraulic Structures	Sept., 1945	
Discussion in Dec., 1945, Feb., Mar., Apr., May, June, Oct., 1946		Closed*
Freudenthal, Alfred M. The Safety of Structures	Oct., 1945	
Discussion in Jan., Feb., Apr., June, 1946		Closed*
Hopkins, Charles D. Balanced Design in Urban Triangulation	Nov., 1945	
Discussion in Apr., Sept., 1946		Closed*
Symposium: Uplift Pressure in and Beneath Dams	Dec., 1945	
Discussion in June, Sept., 1946		Closed*
Symposium: Technical Survey—Brooklyn Bridge After Sixty Years	Jan., 1946	
Discussion in Apr., May, June, 1946		Closed*
Senour, Charles. New Project for Stabilizing and Deepening Lower Mississippi River	Feb., 1946	
Discussion in Sept., 1946		Nov. 1, 1946
Ripperger, E. A., and Davids, N. Critical Stresses in a Circular Ring	Feb., 1946	
Discussion in Oct., 1946		Nov. 1, 1946
Forbes, Hyde. Landslide Investigation and Correction	Feb., 1946	
Discussion in May, Sept., Oct., 1946		Nov. 1, 1946
Winter, George. Strength of Thin Steel Compression Flanges	Feb., 1946	
Discussion in June, Oct., 1946		Nov. 1, 1946
Barnett, Joseph. Express Highway Planning in Metropolitan Areas	Mar., 1946	
Discussion in May, Sept., 1946		Nov. 1, 1946
Lothers, J. E. Torsion in Steel Spandrel Girders	Mar., 1946	
Discussion in June, Oct., 1946		Nov. 1, 1946
Lifsitz, Sol. Analysis of Unsymmetrical Beams by the Method of Segments	Mar., 1946	
Discussion in Oct., 1946		Nov. 1, 1946
Howson, Louis R. Future Costs and Their Effects on Engineering Budgets	Mar., 1946	Nov. 1, 1946
Wood, Charles P. Factors Controlling the Location of Various Types of Industry	Mar., 1946	Nov. 1, 1946
Sette, F. J. The Planning of Aerial Photographic Projects	Mar., 1946	
Discussion in Sept., Oct., 1946		Nov. 1, 1946
Report: Engineering Education: Progress Reports of the Society's Professional Committee on Engineering Education for 1944 and 1945	Mar., 1946	
Discussion in June, Sept., Oct., 1946		Nov. 1, 1946
Dunham, John W. Design Live Loads in Buildings	Apr., 1946	
Discussion in Oct., 1946		Nov. 1, 1946
Peterson, F. G. Eric. Effect of Stress Distribution on Yield Points	Apr., 1946	
Discussion in Sept., 1946		Nov. 1, 1946
Baker, Donald M. Some Thoughts on Engineering Education	Apr., 1946	
Discussion in Sept., Oct., 1946		Nov. 1, 1946
Report: Topographic Surveys: Progress Report of the Committee of the Serveying and Mapping Division on Topographic Surveys	Apr., 1946	
Discussion in Oct., 1946		Nov. 1, 1946
Rose, Edwin. Thrust Exerted by Expanding Ice Sheet	May, 1946	Dec. 1, 1946
Dell, George H. Moment-Stiffness Relations in Continuous Frames with Prismatic Members	May, 1946	
Discussion in Oct., 1946		Dec. 1, 1946
Bermel, Karl J., and Sanks, Robert L. Model Study of Brown Canyon Debris Barrier	May, 1946	Dec. 1, 1946
Jacob, C. E. Effective Radius of Drawdown Test to Determine Artesian Well	May, 1946	Dec. 1, 1946
Report: Opportunities for Sanitary Engineers in the Postwar Period: Progress Report of the Committee on Advancement of Sanitary Engineering of the Sanitary Engineering Division	May, 1946	
Moore, R. L. Observations on the Behavior of Aluminum Alloy Test Girders	June, 1946	Dec. 1, 1946
Hansen, Howard J. Design of Plywood I-Beams	June, 1946	Dec. 1, 1946
Tommerup, Carl C. H. Rigid-Frame Structures Subject to Nonuniform Thermal Action	June, 1946	Dec. 1, 1946
Middlebrooks, T. A., and Jervis, William H. Relief Wells for Dams and Levees	June, 1946	
Discussion in Oct., 1946		Dec. 1, 1946
Report: Advances in Sewage Treatment and Present Status of the Art: Third Progress Report of the Committee of the Sanitary Engineering Division on Sewerage and Sewage Treatment	June, 1946	
Stevens, J. W. Truck Speed and Time Loss on Grades	Sept., 1946	Feb. 1, 1947
Underwood, P. H. Space Resection Problems in Photogrammetry	Sept., 1946	Feb. 1, 1947
McGuinness, Charles L. Recharge and Depletion of Ground-Water Supplies	Sept., 1946	Feb. 1, 1947
de Vries, Karl. Strength of Beams as Determined by Lateral Buckling	Sept., 1946	Feb. 1, 1947

Note.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

CONTENTS FOR OCTOBER, 1946

PAPERS

DISCUSSIONS

CONTENTS FOR OCTOBER, 1946 (*Continued*)

A list of "Current Papers and Discussions" may be found on the page preceding the table of contents

HINTS TO AUTHORS

Those who are planning papers for submission to PROCEEDINGS *will expedite Committee action measurably by first studying the standard instructions as to style, content, and format. For reprints, address the Manager, Technical Publications, 33 W. 39th St., New York 18, N. Y.*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

CORRECTION OF TAILWATER EROSION AT PRAIRIE DU SAC DAM

By C. N. Ward,[1] M. ASCE, and Henry J. Hunt,[2] Esq.

Synopsis

The hydroelectric power plant at Prairie du Sac, Wis., on the Wisconsin River was placed in operation in 1914.[3,4,5] A general lowering of the tailwater and of the river bottom downstream from the plant became obvious a few years later. Because of the presence of a large island immediately below the east half of the spillway section of the dam when constructed, the recession of the tailwater level was not appreciable until 1920 when the island had been completely removed. From then until 1932 the recession of tailwater was more rapid. The average rate of recession of minimum monthly tailwater levels over a 17-year period, 1915–1931, inclusive, was 0.43 ft per yr. Such river controls as an old bridge foundation, two highway bridges, and a railroad bridge and gravel shoals over certain river stretches caused the rate of recession to decrease to 0.10 ft per yr from 1932 to 1936, inclusive, and to practically a stable condition from 1937 to 1944, inclusive. During 1929 hydraulic model tests were made to determine a method of reducing excessive erosion of the river bed immediately downstream from the dam.

Expensive and extensive methods of improvement were not considered in the first study because a new power dam was then under consideration which would, when constructed, restore tailwater levels at Prairie du Sac. Model tests were confined to changes which could be built on the prototype without the need of a cofferdam. A partial or temporary remedy was all that was expected. A wooden baffle was designed on the basis of model tests and was constructed on the prototype in 1930.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **March 1, 1947.**

[1] Partner, Clayton N. Ward-John A. Strand, Cons. Engrs., Madison, Wis.; formerly Partner, Mead, Ward & Hunt, Cons. Engrs., Madison, Wis.

[2] Partner, Mead and Hunt, Cons. Engrs., Madison, Wis.; formerly Partner, Mead, Ward & Hunt, Cons. Engrs., Madison, Wis.

[3] "Construction of the Prairie du Sac Power Plant," *Engineering Record,* May 31, 1913, p. 603.

[4] "Method of Constructing a Hydro-Electric Power House and Dam on Sand Foundation," *Engineering and Contracting,* May 7, 1913, p. 533.

[5] "Building Power House and Dam on Sand Foundation," *Fngineering News,* June 15, 1916, p. 1113.

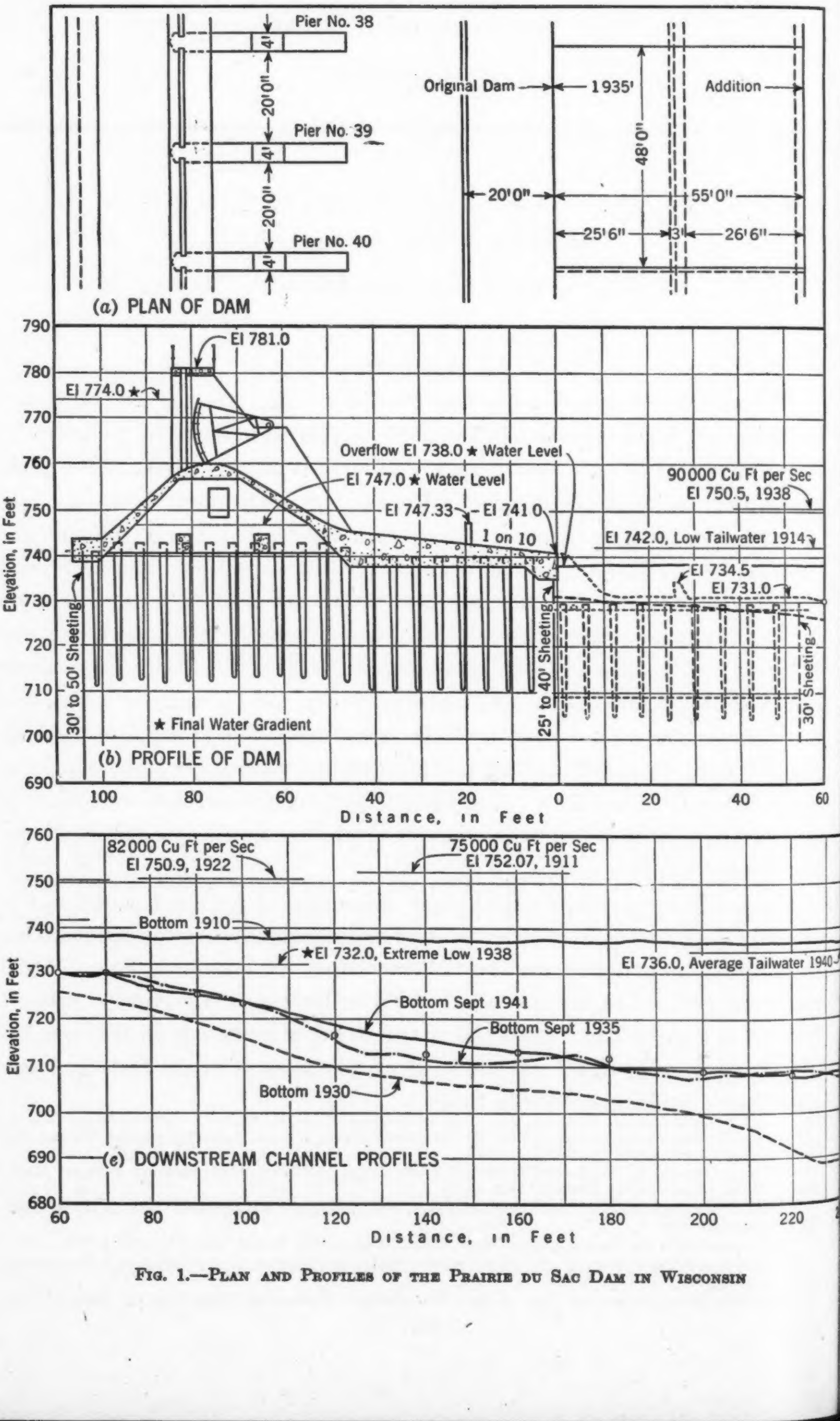
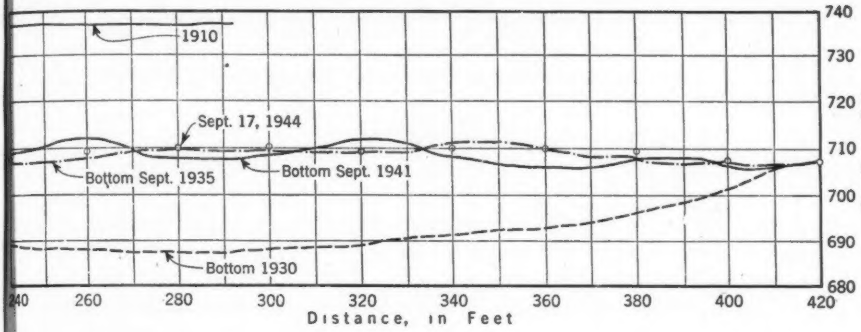


FIG. 1.—PLAN AND PROFILES OF THE PRAIRIE DU SAC DAM IN WISCONSIN



In 1933 it was decided that a new dam would not be constructed in the near future downstream from Prairie du Sac and that works would be designed and constructed at the site to reduce erosion further and to improve other unfavorable conditions arising from lowered tailwater. A new series of tests was then run to determine a basis for design of an improvement which would be complete in itself.

This paper briefly describes the model tests and types of improvements adopted and presents a general comparison of performance of models with that of the prototype.

General Description of Plant

There is an earth embankment about 1,700 ft long on the east side of the river. In the main channel there are forty-one Tainter gates (20 ft wide by 14 ft high) with 4-ft piers, a lock 35 ft wide, a closed dam and sluices 38 ft long, and a powerhouse 329 ft long. Profiles of the river bottom downstream from the dam in a region of maximum bed cutting are shown in Fig. 1. Fig. 2, which was used as a form sheet in recording the direction of water currents and other data in some of the 1933 model tests, is a diagrammatic plan, showing the main structures of the plant and the contours of the river bottom as of 1932. The most serious erosion of the bottom of the river occurred about 220 ft downstream from the end of the original apron below gates Nos. 36 to 41 and near the east side of the lock. The deepest erosion extended 54 ft below the top of the original apron.

TABLE 1.—Tailwater Discharge at Prairie Du Sac, Wis.

Tailwater elevation (ft)	Discharge (Cu Ft per Sec)		
	1912	1929	1931
734	....	....	1,200
735	....	1,500	2,700
736	....	3,250	4,250
737	....	5,500	6,000
738	....	8,100	8,300
739	....	11,000	11,100
740	....	14,400	14,400
741	3,500	18,000	18,000
742	5,750	21,800	21,800
743	8,250	26,000	26,000
744	11,000	30,500	30,500
745	14,250	35,250	35,250
746	17,700	40,500	40,500
747	22,000	46,300	46,300
748	27,750	54,200	54,200
749	34,750	65,000	65,000
750	43,000	....	....

"Tailwater discharge" curves are shown for the period from 1909 to 1931 in Table 1. During this time the tailwater receded about 5.8 ft for a flow of 5,000

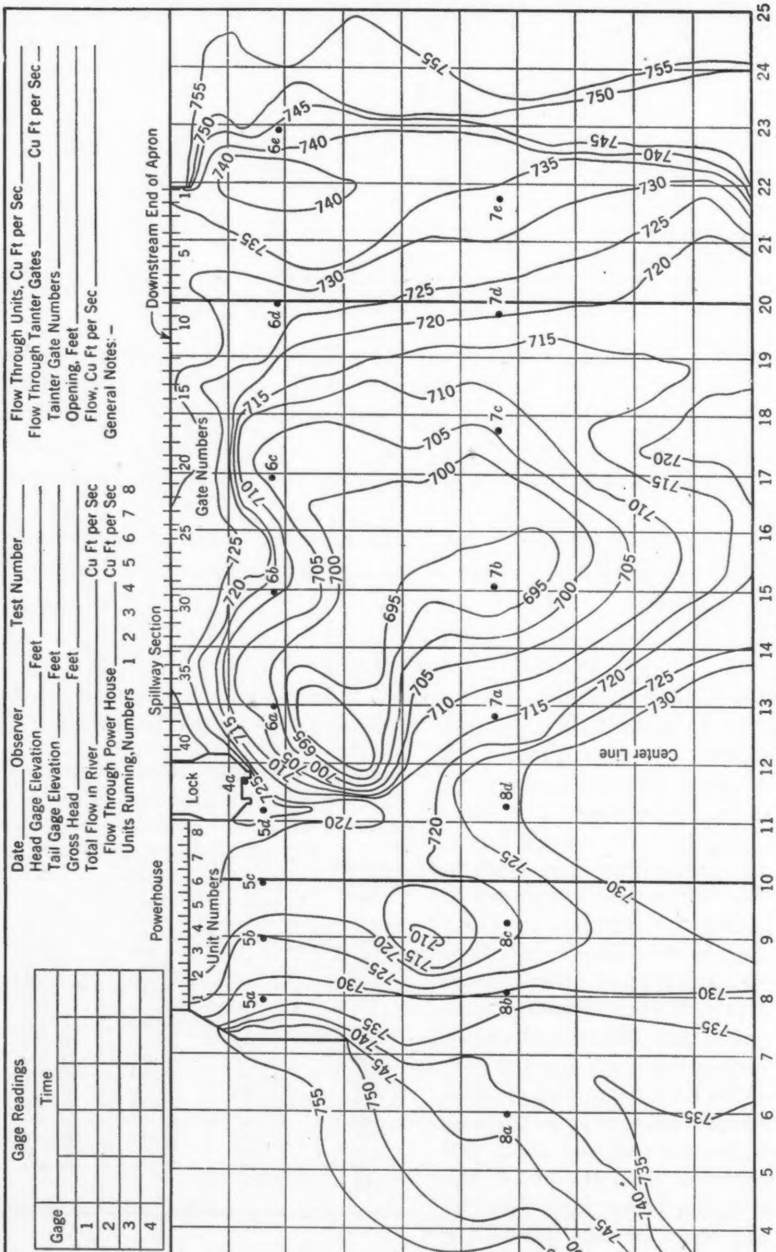


FIG. 2.—FORM SHEET AND GENERAL CONTOUR PLAN OF RIVER BOTTOM AS OF 1932

cu ft per sec and 3.3 ft for a flow of 50,000 cu ft per sec. The actual period of recession began in 1914 so that the rate for 5,000 cu ft per sec was 0.34 ft per yr; and, for 50,000 cu ft per sec, the rate was 0.20 ft per yr.

1929 Tests

A model of three gates of the dam to a scale of 1 to 20 was used in the 1929 tests at the Hydraulic Laboratory of the University of Wisconsin at Madison. The dam, constructed of wood, was tested in a wooden box 30 ft long, which had glass sides permitting visual inspection of flow conditions on the apron of the dam for a distance of 70 ft (on prototype) downstream from the end of the apron. All measurements in the tests were recorded and are referred to in terms of units applicable to the prototype. Two venturi meters (4 in. and 8 in.) were used to measure the flow of water. Water was admitted to the space underneath the dam and apron which acted as a stilling basin. Baffles caused the water to flow to the gates smoothly. A gate used at the downstream end of the box to control tailwater levels was designed to allow water to discharge so that normal vertical velocity curves would occur a short distance upstream from the gate.

The control gate was of the register type consisting of two wooden members, each having rectangular orifices, all the same height and arranged in horizontal rows with different spacing at various depths. One member was fixed; and the other was placed in contact with it and was vertically adjustable. The orifices were designed to discharge water at different levels, thus giving a normal vertical velocity distribution in the model for the average conditions to be tested. The gate was found to be very effective in controlling tailwater heights, and velocity measurements made with a pitot tube showed that normal and desired velocity distribution actually occurred a short distance upstream from this gate.

TABLE 2.—Sieve Analysis of Sands from Prototype and of Sand Used in Model

Sieve No.	Percentage Retained (Weight)[a]						
	Sand No. 1	Sand No. 2	Sand No. 3	Sand No. 4	Sand No. 5	Sand No. 6	Sand No. 7
8	0	0	0	0	6.5	0	0
14	2.2	1.0	1.5	2.2	20.5	0	0.5
28	19.0	2.5	8.5	9.3	35.0	0	1.1
48	94.0	40.0	52.5	91.5	76.0	22.5	6.0
100	100	94.0	98.3	100	99.0	82.0	18.5
150	....	99.5	99.5	....	100	95.0	26.5
200	....	100	100	....	....	97.5	30.5
Pan	....	....	....	....	....	100	100

[a] The samples of the sands were taken at the following places: No. 1, center of river below locks; No. 2, near west shore; No. 3, near east shore; No. 4, center of tailrace near island; No. 5, Janesville, Wis.; and Nos. 6 and 7, model sands, at the Wisconsin foundry.

The bottom of the box was filled with sand and the surface of the sand was made to conform with various profiles of the river bottom. In some tests the bottom was shaped to conform to the river bottom downstream from gate No. 9 and in others to an average profile of the river bottom.

Since the exact laws of similitude relating to movement of soil grains by water were not known, it was decided to use the finest sand readily attainable for the bottom of the channel of the model. Sieve analyses of sand from the prototype are shown in Table 2, sands Nos. 1, 2, 3, and 4, and the sand used in the model is designated as sand No. 6. Sand No. 7 was rejected because it contained a large amount of fine material that was carried in suspension with a very slight movement of water. It was decided that a study of bed movement in the model would be relatively indicative of results attainable in the prototype even though the exact quantitative relationship was not known.

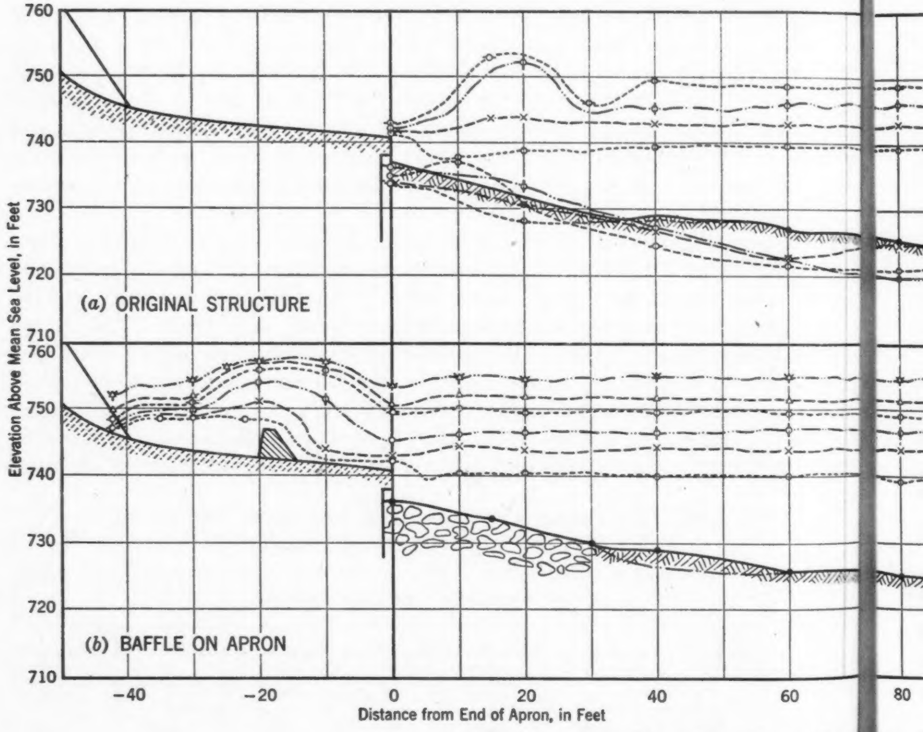


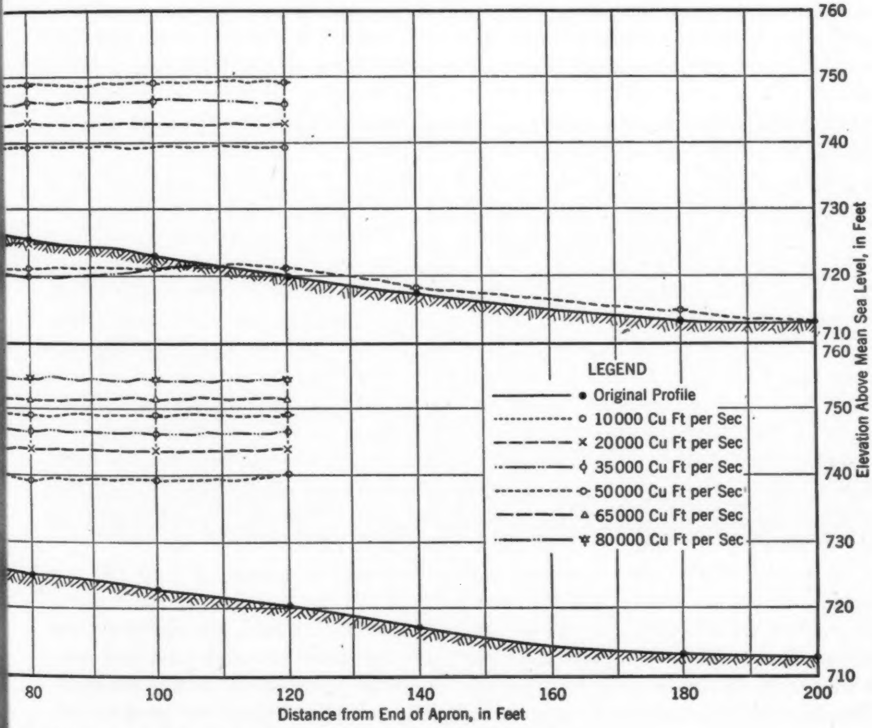
FIG. 3.—WATER SURFACE AND RIVER BOTTOM PROFI FROM T

Attempts to measure water velocities in the model with a small current meter designed for this purpose and a pitot tube were not satisfactory in the 1929 tests. Yarn tied on small sticks was used to determine the presence and direction of water currents. Direct observations through the glass windows of the model indicated the general magnitude of bottom velocities since the movement of soil particles could be seen.

Early tests showed that the sand in the model reached a stable condition in about 1 hour after a run was started. Each of thirty-four different arrangements of the model were tested under from four to six rates of flow varying from

10,000 cu ft per sec to 80,000 cu ft per sec. Tailwater rating curves used in tests usually followed the 1924 rating curve.

Water profiles and sand bottom profiles were taken during and after each run. Fig. 3(*a*) shows water profiles and bottom profiles in one test with the original dam; and Fig. 3(*b*), a similar test with the exception of a 52-in. baffle wall placed on the apron of the dam. The sand lost from the bottom in the first 200 ft downstream from the dam for flows up to 50,000 cu ft per sec was 270 cu yd per gate (in terms of the prototype), for the dam as originally constructed, whereas no measurable loss resulted with the same range of flows when



FROM TESTS OF A 1-to-20 SCALE MODEL (1929 TESTS)

the 52-in. baffle was used. A baffle 52 in. high, 20 ft from the downstream end of the apron, appeared to improve conditions materially. Such a baffle was then constructed on the prototype below two Tainter gates. Because of the river flow conditions, the tailwater could not be built up to flood stages. However, the two gates were opened to discharge at rates equivalent to a flood flow of 30,000 cu ft per sec in the river. Flow conditions over the baffle appeared generally similar to those observed on the model. The pool in front of the baffle was maintained on the prototype as in the model. No movement of river bed material downstream from the gates occurred.

A baffle 52 in. high was built across the entire length of the prototype dam in 1930. It was constructed with 8-in. timbers used as stop logs supported by 12-in. H-beams placed in the apron of the dam. From 1930 to 1933 this baffle improved conditions materially.

1933 Tests

It was definitely decided in the early part of 1933 that a new dam downstream from Prairie du Sac would not be constructed in the near future. Because of recession of the tailwater, the Prairie du Sac Dam would have been subjected to a head considerably greater than that for which it was originally designed if normal headwater elevation were maintained. For a number of years the plant was operated with the headwater lowered to reduce the head on the dam. Deep holes, eroded in the river bottom prior to 1930, had not been filled. Thus, a new series of tests was run to develop plans for the permanent improvement of the structures at Prairie du Sac that would remedy the unfavorable conditions resulting from receding tailwater elevations.

After a brief study, it was decided to design new works which would be constructed adjacent to and downstream from the end of the Prairie du Sac apron—itself able to withstand a head of 13 ft and to maintain a downstream head on the old dam up to about El. 742. The model tests made in 1930 indicated quite clearly the procedure required to prevent erosion of the sand bottom. It was felt desirable, however, to make further studies using a model which incorporated the entire dam lock and powerhouse. Observations of the prototype indicated that serious crosscurrents and eddying were caused when currents of different velocities joined downstream from the dam and the powerhouse. The discharge from certain combinations of Tainter gates resulted in obvious disturbances. Eddying also appeared downstream from the lock, where water discharged through the gates and water discharged through the powerhouse joined. A survey was made of the river channel downstream from the Prairie du Sac Dam for a distance of about a mile.

A model of all structures and of the river bed to a scale of 1 to 100 was made outdoors near the powerhouse at Prairie du Sac. Accurate sheet metal templets were cut to the shape of the river channel at each change of section of the river. These templets were supported on short wooden piles and were held firmly by bolts placed to permit accurate adjustment of the templets. Concrete was then poured between templets and the surface was struck with a straight edge. Great care was taken to reproduce, accurately, the channel of the prototype. All details of the dam and the power plant which concerned river flow were carefully reproduced in the model. A gate at the downstream end of the model which controlled the tailwater consisted of a plank hinged at the bottom with its top cut to conform generally to the shape of the section of the river. In whatever position it was placed, the gate allowed water to be released from the model so that the distribution of flow across the section was very nearly the same as that in the prototype. The model of the river bottom was poured with concrete composed of white cement and light-colored sand. The concrete bottom was divided into 1-ft squares by coordinates painted with asphalt. One coordinate was parallel to the end of the apron of the dam.

Ten different arrangements (a total of one hundred and six runs) of the model were tested under various flood flows with different distributions of flow through the powerhouse and over the dam and with various combinations of gates to discharge flood flows. Fig. 4 shows one view of the 1-to-100 scale model. Average vertical velocities in the model were studied with small vertical floats loaded so that they would float with one end slightly exposed. Floats were started near the end of the original apron of the dam. One observer plotted the path of each float on the form sheet shown in Fig. 2 by observing the movement of the float relative to the reference lines painted on the surface of the model. Another observer with a stop watch noted the time at which each float crossed the reference lines. These time observations were recorded on the sheet. Floats were selected so that they would just clear the bottom of the channel. Whenever a float encountered shallower water and dragged

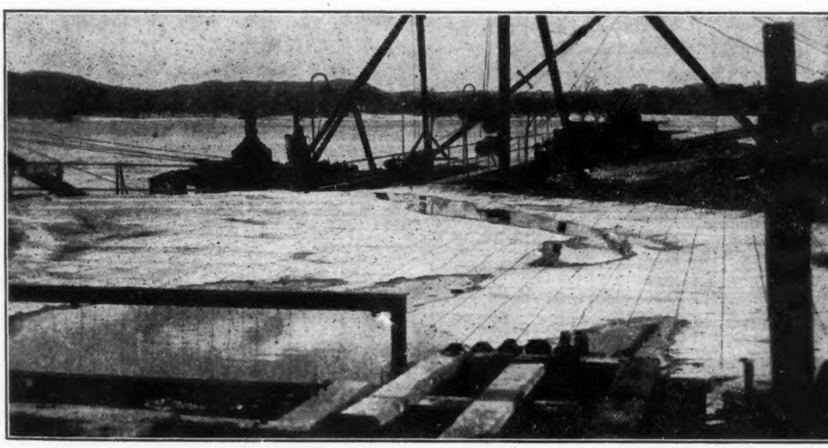


FIG. 4.—MODEL OF THE PRAIRIE DU SAC PLANT; SCALE, 1 to 100

on the bottom, a shorter float was started a short distance upstream from the obstruction and observations of velocities were continued. Bottom velocities were observed with wooden spheres about $\frac{1}{4}$ in. in diameter which were loaded so that they would sink very slowly in still water. The course followed by each ball and the time were recorded as for the vertical staff floats.

It was decided that a new apron 55 ft wide would be constructed on the prototype at the downstream end of the old apron with the top elevation at El. 731 or 10 ft below the top of the end of the original apron. The 1929 tests indicated that the baffle placed on the old apron would be effective with the new apron. Tests were then run on the 1-to-20 scale model (which had been used in the 1929 tests) to determine the effect of a baffle wall on the new apron and to verify conclusions reached in the studies with the 1-to-100 scale model. Nine different conditions were studied on the 1-to-20 scale model; and it was found that there was no erosion in the bed of the model with: (*a*) The new apron constructed at El. 731; (*b*) the baffle on the old apron as built in 1930; or (*c*) an

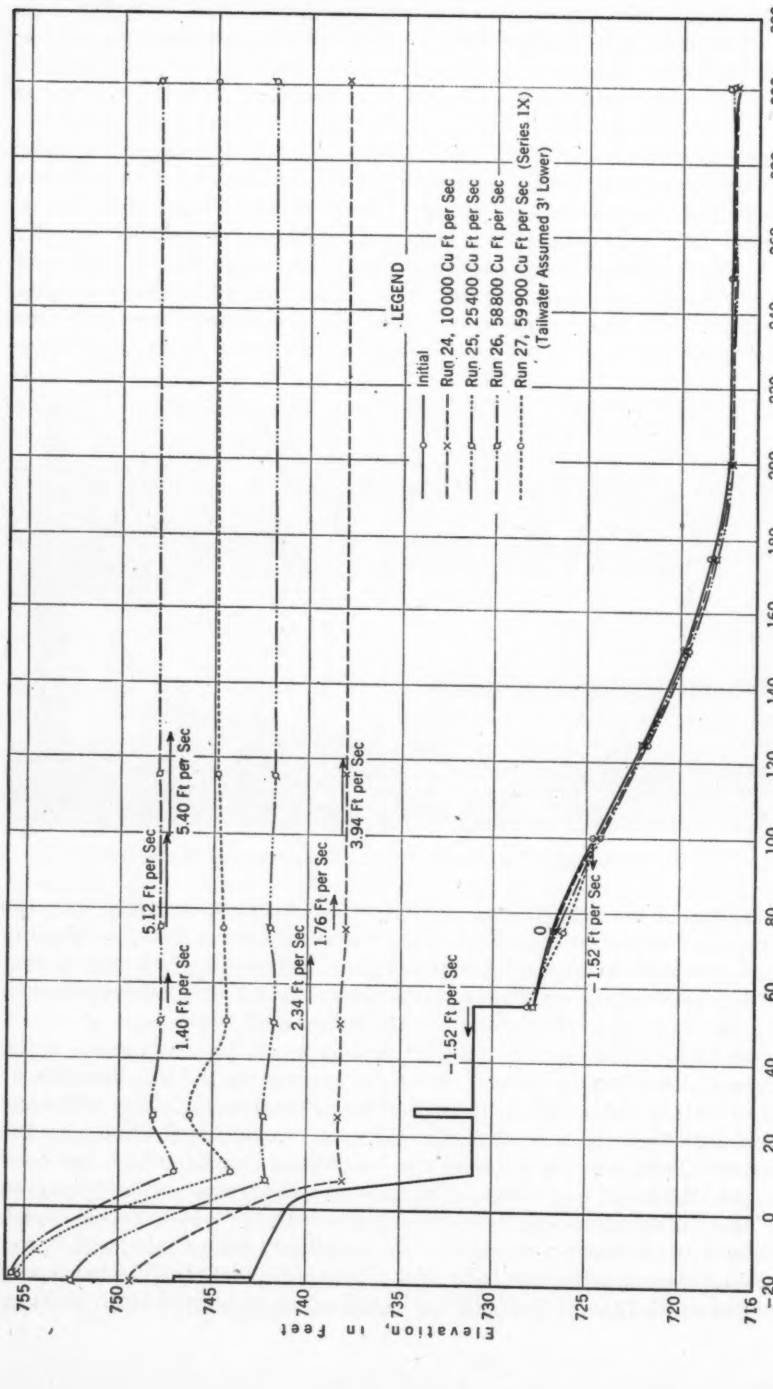


FIG. 5.—TESTS IN 1933; 1-TO-20 SCALE MODEL WITH NEW APRON AND BAFFLES

additional baffle 3.5 ft high on the new apron about 30 ft from the downstream end of the new apron.

Tests with the 1-to-100 scale model with the new apron at El. 731 were made with the depressions in the river bottom all filled to El. 710, and bottom velocities were found satisfactory under this condition. Fig. 5 shows the results of the test of the 1-to-20 scale model for the arrangement which was finally adopted with various flows and with tailwater elevations in accordance with the 1929 rating curve. In this test, erosion of the river bed was very small.

The velocities of flow were measured in the model with a pitot tube at the end of the new apron, 20 ft downstream and 45 ft downstream, for surface, midsection, and bottom. In Fig. 5 one set of these velocities for run No. 26, series VIII, 58,800 cu ft per sec, shows the relative positions of observations and the magnitude of each. A tabulation of the velocities for the other runs shown in Fig. 5 is given in Table 3.

TABLE 3.—VELOCITY MEASUREMENTS (FEET PER SECOND) IN A 1-TO-20 SCALE MODEL

(Reduced to Prototype Values)

Location	Downstream Distance, in Feet, from End of Apron		
	0	20	45
(a) Run No. 24; Series VIII; 10,000 Cu Ft per Sec			
Surface	+2.33	+3.64	+2.16
Middle	+1.53	+1.98	+1.53
Bottom	+1.76	0	0
(b) Run No. 25; Series VIII; 25,400 Cu Ft per Sec			
Surface	+3.80	+3.64	+1.97
Middle	+3.42	0	+2.33
Bottom	+2.93	+1.98	−2.97
(c) Run No. 26; Series VIII; 58,800 Cu Ft per Sec			
Surface	+1.40	+5.12	+5.40
Middle	+2.34	+1.76	+3.94
Bottom	−1.52	0	−1.52
(d) Run No. 27; Series IX; 59,900 Cu Ft per Sec; Tailwater Assumed 3 Ft Below Normal for Same Flow			
Surface	+9.30	+5.23	+4.68
Middle	+4.85	+3.05	+2.55
Bottom	+2.34	−1.77	+1.77

A new apron was constructed on the prototype in 1935 at El. 731 with a baffle at El. 734.5 and with a smooth transition section from El. 741 to the top of the new apron.

Figs. 6(*a*) and 6(*b*) are photographs of the model with the new apron and baffles which were finally adopted. Fig. 6(*a*) shows the baffles and apron of the model; and Fig. 6(*b*), a flood flow of 58,800 cu ft per sec on the model A flood of 90,000 cu ft per sec occurred on September 15, 1938, and was the highest recorded flood at Prairie du Sac. The second largest flood—82,000 cu ft per sec—occurred on April 14, 1922.

The new apron was designed with a water seal between it and the old apron so that either structure could move somewhat without causing a reaction on the other. Sheet piling was placed at the downstream end of the new apron. The apron was supported on wood bearing piles placed to withstand uplift—so that the new structure would safely withstand a head of 13 ft or any head up to El. 743. A drain was constructed of porous concrete under the new apron which discharged normally at El. 738.0 into manholes at the ends of the dam. Stop logs were provided in the manholes which permitted adjustment of the water level under the new structure. The hydraulic gradient on a section

across the spillway structure would be: Headwater elevation, 774.0 ft; elevation within the hollow spillway section, 747.0 ft; elevation for drainage overflow at new apron, 738.0 ft; and minimum tailwater elevation, 732.0 ft.

The line labeled "September, 1935," in Fig. 1, represents the profile of the river bottom as it was just after construction of the apron on the prototype. The solid line shows the position of the river bottom in 1941; and the circles, the bottom in 1944. There has been no loss of material in the first 200 ft downstream from the end of the old apron. In fact, the river bottom has been raised somewhat by deposition of material brought upstream from points 400 ft

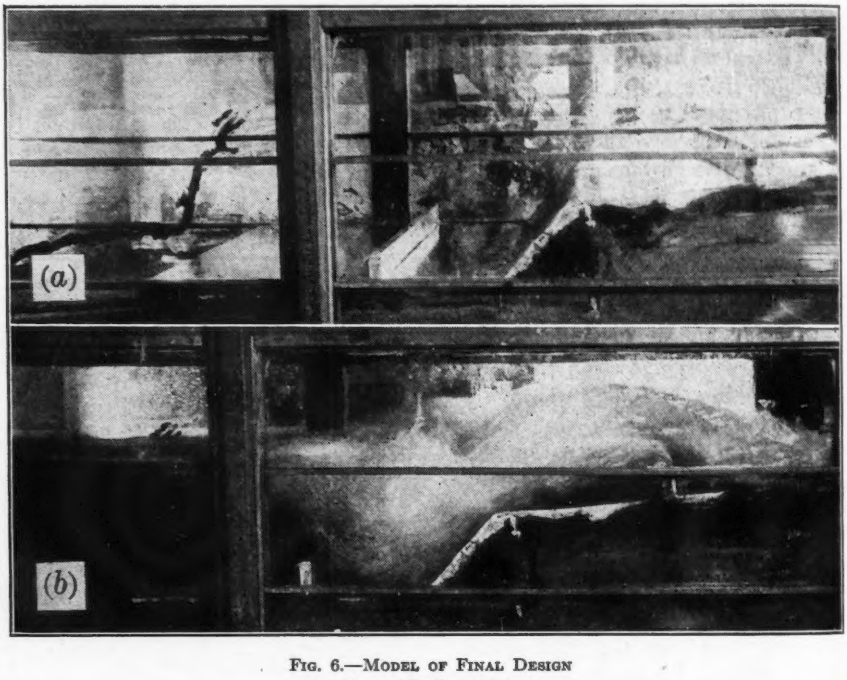


Fig. 6.—Model of Final Design
(a) View of Baffles and Apron
(b) Flow, 58,800 Cu Ft per Sec

or more from the downstream end of the apron. The condition at this section is quite typical of that at all sections. At gate No. 7 a slight amount of material was removed between points 220 ft and 400 ft downstream from the end of the old apron—the average removal being about $2\frac{1}{2}$ ft and the maximum 5 ft. In this region where there was a slight loss of material, the bottom had been left somewhat above the general bottom elevation of adjacent sections at the time of construction. In general, there has been no loss of material except in very small isolated areas which were not leveled off to conform with general elevations of surrounding areas. The new construction has been subjected to a greater range of flow conditions than had occurred within the period of record

prior to the construction of the new apron. The greatest flow ever recorded passed the river at Prairie du Sac at 8 a.m. on September 15, 1938. Soundings have been taken every year since the apron was completed; and there has never been any material change in the river bottom since 1935—except tendencies toward general raising of the river bottom in the first 200 ft downstream from the old apron and toward leveling off high spots which were left after construction.

Both the 1-to-100 scale model tests and the 1-to-20 scale model tests indicated that removal of bed material below the dam was caused by upstream currents which existed from a distance at least 180 ft downstream from the end of the old apron to the apron. In no case did the jet from the old apron plunge under causing high bottom velocities in a downstream direction. Excessive bottom cutting in the model was always caused by upstream currents which occupied about two thirds of the depth of the channel.

Conclusions

At various times during the tests of the 1-to-100 scale model at Prairie du Sac, the model was adjusted to simulate conditions then existing on the prototype. Both the model and the prototype could be observed at one time. Under one condition, there was a pronounced surface current in the prototype which began about 1,200 ft downstream from the powerhouse and extended across the channel upstream toward the center of the Tainter gate section of the dam. With comparable flow distribution and tailwater level, such a condition was found to exist in the model. In a similar manner various combinations of powerhouse units that caused eddies in the tailrace were duplicated on the model. The distribution and direction of currents in the two sizes of models were nearly alike. High-velocity upstream currents on the river bottom occurred over essentially the same areas in the two models.

In the model tests the new apron and baffles reduced bottom velocities near the dam under all conditions—the highest bottom velocities were small and were generally in an upstream direction. The baffle on the old dam used with the new apron gave good results; but a material improvement was experienced when the baffle was placed on the new apron.

The design of improvements, based on model tests, has proved entirely satisfactory. The profiles of river bottoms in Fig. 1 show that bottom velocities have been reduced so that there is no objectionable cutting of the river bed near the dam.

High bottom velocities and considerable river bed movement occurred in the models under conditions which caused serious erosion in the prototype. Ten years of experience with the new structures indicate that the new apron and baffles were as effective in eliminating erosive velocities in the prototype as they were in the model.

A

pl
Th
als
an
tio
fo
su
is
me
of

ing
sin
cec
Sin
the
gin
eva
lar
ope
nu
mu

cuss

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

MATRIX ANALYSIS OF CONTINUOUS BEAMS

By Stanley U. Benscoter,[1] Assoc. M. ASCE

Synopsis

An introduction to the use of matrix algebra in the development and explanation of various methods of structural analysis is presented in this paper. The possibilities of making rapid design calculations by matrix methods are also illustrated. The paper is divided into four parts: In the first part an explanation is given of the elementary operations of addition, subtraction, multiplication, and division of matrices; in the second part a general formula is derived for the solution of any continuous beam (on unyielding supports) in matrix notation; in the third part the moment distribution process is developed by matrix algebra; and in the fourth part two successive correction methods of computation which may be regarded as variations of the method of moment distribution are illustrated and discussed.

PART I. ELEMENTARY OPERATIONS

Introduction

Matrix algebra may be regarded as a "shorthand" technique for representing a system of linear equations by a single equation and then solving that single equation. The rules of matrix algebra provide a computational procedure which is often more rapid than the numerical processes in common usage. Since all indeterminate structures are governed by systems of linear equations, the possibility of useful application of matrix methods by the structural engineer is suggested. The ability of the reader to understand, appreciate, and evaluate the matrix methods presented in this paper will be dependent, in a large measure, on his willingness to familiarize himself with the elementary operations. These mechanical procedures can be learned only by practice on numerical examples, just as the mechanical procedure of moment distribution must be learned.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by February 1, 1947.

[1] Aeronautical Engr., Bureau of Aeronautics, Washington, D. C.

Evaluation of Determinants

It is essential that the reader recall the process of evaluating a determinant. The determinant of A is a square array of numbers which is written in the following form:

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \text{.......... (1)}$$

In order that the reader may check his ability to evaluate determinants, the following examples are given of second-order and third-order determinants:

$$\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix} = 12 - 2 = 10 \quad \text{.......... (2a)}$$

and

$$\begin{vmatrix} 5 & 1 & 2 \\ 0 & 3 & 1 \\ 1 & 2 & 4 \end{vmatrix} = 5 \times 10 - 1 \times (-1) + 2 \times (-3) = 45 \quad \text{..... (2b)}$$

Addition and Subtraction of Matrices

A matrix is also an array of numbers, but it may be either square or rectangular. A matrix will be indicated by a bracket and written in the following form:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{.......... (3)}$$

The subscripts of the elements of the matrix or determinant are chosen so that the first subscript corresponds to the row containing the element and the second subscript corresponds to the column. To make use of matrices it is necessary to know the rules for addition, subtraction, multiplication, and division of matrices. These rules will be stated as simply as possible and should be learned by application to numerical cases.

Consider the matrix $[A]$ in Eq. 3 and the matrix $[B]$ given in the following form:

$$[B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \quad \text{.......... (4)}$$

If the matrices $[A]$ and $[B]$ are added, a matrix $[C]$ is obtained and $[C]$ is also a 3×3 array of numbers:

$$[A] + [B] = [B] + [A] = [C] \quad \text{.......... (5)}$$

in which

$$[C] = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \quad \text{.......... (6)}$$

Each element of $[C]$ is obtained by adding the corresponding elements of $[A]$

and $[B]$. Thus, the elements of $[C]$ are computed as follows:

$$c_{11} = a_{11} + b_{11} \quad \text{(7a)}$$

$$c_{12} = a_{12} + b_{12} \quad \text{(7b)}$$

$$c_{13} = a_{13} + b_{13} \quad \text{(7c)}$$

etc. Matrices are subtracted in the same manner.

The following examples illustrate the processes of addition and subtraction; let

$$[M] = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}; \text{ and } [N] = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} \quad \text{(8)}$$

then

$$[S] = [M] + [N] = \begin{bmatrix} 3 & 1 \\ 3 & 7 \end{bmatrix} \quad \text{(9)}$$

and

$$[T] = [M] - [N] = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \quad \text{(10)}$$

Multiplication of Matrices

If the matrices $[A]$ and $[B]$ are multiplied, a matrix $[D]$ is obtained which is also a 3×3 array of numbers:

$$[A][B] = [D] \quad \text{(11)}$$

in which

$$[D] = \begin{Bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{Bmatrix} \quad \text{(12)}$$

The elements of $[D]$ are computed from the elements of $[A]$ and $[B]$ by using the "row by column" rule. The rows of $[A]$ are multiplied by the columns of $[B]$. The row of $[A]$ to be used corresponds to the row of the element of $[D]$ being computed and the column of $[B]$ to be used corresponds to the column of the element of $[D]$. Thus, d_{11} is computed by multiplying the first row of $[A]$ by the first column of $[B]$. This multiplication may be indicated by

$$d_{11} = [a_{11}\ a_{12}\ a_{13}] \begin{Bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{Bmatrix} \quad \text{(13)}$$

These quantities are multiplied by adding the products of corresponding elements:

$$d_{11} = a_{11}\,b_{11} + a_{12}\,b_{21} + a_{13}\,b_{31} \quad \text{(14)}$$

A curious but important property of matrices is that they are noncommutative in multiplication. Thus, in general, the product $[A][B]$ does not equal the product $[B][A]$. This property of matrices deserves emphasis since the structural engineer is not accustomed to dealing with noncommutative numbers.

The following example illustrates the process of multiplication; let

$$[A] = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}; \text{ and } [B] = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

then

$$[A][B] = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 13 & 6 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (16a)$$

and

$$[B][A] = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 7 & 4 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (16b)$$

Eqs. 16 show how the reversal of the order of multiplication gives a different answer. Two more examples will illustrate the multiplication of rectangular matrices:

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 20 \\ 3 & 7 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

In Eq. 17 a 3 $\times$ 2 matrix is multiplied by a 2 $\times$ 3 matrix and the result is a square 2 $\times$ 2 matrix. In Eq. 18, a 3 $\times$ 2 matrix is multiplied by a column matrix, commonly called a column vector. The result is a column vector of two elements. The elements of a column vector are frequently called the components of the vector. The reader should attempt to multiply the matrices in Eqs. 17 and 18 in reverse order. In the first case the multiplication is possible whereas in the second case it is impossible. The first matrix in a multiplication must have the same number of columns as there are rows in the second matrix.

If a matrix is to be multiplied by a constant, each element of the matrix must be multiplied by the constant. Thus,

$$2\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 0 & 6 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Division of Matrices

To explain the process of dividing matrices, it is necessary to introduce the following definitions which are applicable only to square matrices:

1. The "principal diagonal" of a matrix is the set of elements extending from the upper left-hand corner to the lower right-hand corner.

2. The "transpose" of $[A]$ is obtained by rotating $[A]$ about its principal diagonal. The rows and colums thus become interchanged. A transpose is indicated by a prime. Thus,

$$[A] = \begin{bmatrix} 2 & 7 \\ 3 & 1 \end{bmatrix}; \text{ and } [A]' = \begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots (20)$$

3. A matrix is "symmetrical" if the matrix is equal to its transpose. Thus,

$$[A] = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}; \text{ and } [A] = [A]' \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

4. Corresponding to each element of a matrix (or determinant) there is a "minor" which is obtained by striking out the row and column of the matrix (or determinant) containing the element. The minor corresponding to an element of a matrix (or determinant) is a determinant that can be evaluated to obtain an ordinary number. Considering the determinant of Eq. 1 or the matrix of Eq. 3, the following minors illustrate the definition:

$$\text{Minor corresponding to } a_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \quad \ldots\ldots\ldots\ldots (22a)$$

and

$$\text{Minor corresponding to } a_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \quad \ldots\ldots\ldots\ldots (22b)$$

5. The "cofactor" of an element of a matrix (or determinant) is the signed minor corresponding to that element. A "signed minor" is a minor with the appropriate sign, either plus or minus, prefixed. The proper sign for the cofactor of the first element a_{11} is positive. The signs for the cofactors of the other elements alternate according to their position relative to a_{11}. Thus, the cofactor of a_{12} is negative, that of a_{13} is positive, that of a_{21} is negative, etc. It is convenient to use a corresponding capital letter to indicate a cofactor. Thus,

$$\text{Cofactor of } a_{11} = A_{11} = + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \quad \ldots\ldots\ldots\ldots (23a)$$

and

$$\text{Cofactor of } a_{12} = A_{12} = - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \quad \ldots\ldots\ldots\ldots (23b)$$

6. The "adjoint" of a matrix $[A]$ is the matrix obtained by replacing each element of $[A]$ by its cofactor and then transposing the matrix. Thus,

$$\text{Adjoint of } [A] = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots (24)$$

The following numerical example is an illustration:

$$[A] = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 1 \\ 2 & 4 & 3 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

$$\text{Adjoint of } [A] = \begin{bmatrix} 2 & -9 & 3 \\ -1 & 6 & -2 \\ 0 & -2 & 1 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (26)$$

7. The "unit matrix" is a matrix in which the elements on the principal diagonal are 1's whereas the remaining elements are 0's. It is indicated by

$[I]$; thus,

$$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \ldots\ldots (27)$$

This matrix is also called the identity matrix. It plays a part in the algebraic manipulation of matrices which is similar to the part played by unity in the algebra of ordinary numbers. With ordinary numbers:

$$1 \times a = a \times 1 = a \ldots\ldots (28)$$

With matrices:

$$[I][A] = [A][I] = [A] \ldots\ldots (29)$$

The verity of Eq. 29 can be easily demonstrated by applying the "row by column" rule to a numerical example.

8. The "reciprocal" (or "inverse") of $[A]$ is a matrix which, when multiplied by $[A]$, produces the unit matrix. Thus,

$$[A][A]^{-1} = [A]^{-1}[A] = [I] \ldots\ldots (30)$$

The following example can be checked by the "row by column" rule for multiplication:

$$[A] = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix}; \text{ and } [A]^{-1} = \begin{bmatrix} 4/3 & -1/3 \\ -5/3 & 2/3 \end{bmatrix} \ldots\ldots (31)$$

Multiplying $[A]$ by $[A]^{-1}$ gives:

$$[A][A]^{-1} = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 4/3 & -1/3 \\ -5/3 & 2/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ldots\ldots (32)$$

The foregoing definition of a reciprocal gives no clue as to how it can be obtained. The formula for computing the reciprocal of a matrix is as follows:

$$[A]^{-1} = \frac{\text{Adjoint of } [A]}{|A|} \ldots\ldots (33)$$

The reciprocal is equal to the adjoint divided by the determinant. Considering the example of Eq. 31,

$$\text{Adjoint of } [A] = \begin{bmatrix} 4 & -1 \\ -5 & 2 \end{bmatrix} \ldots\ldots (34)$$

and

$$|A| = 2 \times 4 - 1 \times 5 = 3 \ldots\ldots (35)$$

Substituting into Eq. 33, the reciprocal is obtained as given in Eq. 31.

The process of division can now be explained. If $[B]$ is to be divided by $[A]$, the result may be obtained by multiplying $[B]$ by $[A]^{-1}$. This multiplication can be performed in two ways: $[B]$ can be premultiplied by $[A]^{-1}$ as indicated by $[A]^{-1}[B]$, or it can be postmultiplied by $[A]^{-1}$ to obtain $[B][A]^{-1}$. In general, these two methods of multiplication will give two different results. Consequently, it should be apparent that a representation of

division by writing a fraction, such as $\frac{[B]}{[A]}$, would be ambiguous and hence not permissible. Whenever a matrix equation is to be divided by $[A]$, either every term in the equation must be premultiplied by $[A]^{-1}$, or every term in the equation must be postmultiplied by $[A]^{-1}$.

Solution of Simultaneous Equations

The usefulness of matrix algebra in solving simultaneous equations can now be explained. Consider the following system of three equations in three unknowns:

$$3\,x_1 + x_2 - 2\,x_3 = 2 \quad \text{(36a)}$$

$$x_1 + 2\,x_2 + 3\,x_3 = 1 \quad \text{(36b)}$$

$$2\,x_1 - x_2 + 4\,x_3 = 3 \quad \text{(36c)}$$

Eqs. 36 can be expressed, by using matrices, in the following manner:

$$\begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \quad \text{(37)}$$

If the matrices are multiplied, and corresponding elements on each side of the matrix equation are then equated, the original Eqs. 36 will be reproduced.

Introduce the following definition:

$$[A] = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{bmatrix} \quad \text{(38a)}$$

with

$$[x] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}; \text{ and } [k] = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \quad \text{(38b)}$$

Eq. 37 becomes

$$[A][x] = [k] \quad \text{(39)}$$

The original system of linear algebraic equations has now been reduced to a single linear matrix equation. Eq. 39 is solved by multiplying through by $[A]^{-1}$. Thus,

$$[A]^{-1}[A][x] = [A]^{-1}[k] \quad \text{(40a)}$$

or

$$[I][x] = [A]^{-1}[k] \quad \text{(40b)}$$

or

$$[x] = [A]^{-1}[k] \quad \text{(40c)}$$

Eq. 40*c* gives the values of the components of the column vector $[x]$ by a simple multiplication of matrices, provided $[A]^{-1}$ can be determined. Following the rules and definitions previously given, $[A]^{-1}$ can be computed as

$$[A]^{-1} = \frac{1}{45}\begin{bmatrix} 11 & -2 & 7 \\ 2 & 16 & -11 \\ -5 & 5 & 5 \end{bmatrix} \quad \text{(41)}$$

Eq. 40c becomes

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{45} \begin{bmatrix} 11 & -2 & 7 \\ 2 & 16 & -11 \\ -5 & 5 & -5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \qquad (42)$$

By performing the multiplication indicated on the right, the following values of x_1, x_2, and x_3 can be obtained:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 41/45 \\ -13/45 \\ -4/9 \end{bmatrix} \qquad (43a)$$

or

$$x_1 = \frac{41}{45}\,;\; x_2 = \frac{-13}{45}\,;\text{ and } x_3 = \frac{-4}{9} \qquad (43b)$$

Since continuous structures are governed by linear algebraic equations, it should be apparent that matrix algebra may provide a useful technique for dealing with certain types of structures.

PART II. GENERAL MATRIX FORMULAS FOR CONTINUOUS BEAMS

Introduction

An analysis of a continuous beam will be developed in matrix notation on the assumption that the supports do not deflect. If the reactions of a continuous beam can be determined, the values of bending moments and shears at various sections can be calculated by statics; or, if the bending moment over each support can be determined, the moments and shears at various sections can likewise be determined. Many methods of analysis of continuous beams have been developed. In each method certain fundamental quantities appear which must be determined for each span individually. These fundamental quantities are dependent on the physical properties of the beam and on the loads in the spans. Consequently, it is necessary first to consider an individual span. The algebraic relationships between the fundamental quantities that appear in various methods of continuous beam analysis have been shown by Fang-Yin Tsai.[2]

Column Analogy

A development of the properties of a single span may begin by considering either a simply-supported beam or a fixed-end beam. The latter case will be used. Any complete group of characteristics of a fixed-end beam can be expressed in terms of five fundamental properties. Three of these properties are functions of the physical shape of the beam and two of them are dependent on both the beam shape and the loading. The column analogy method,[3] which was introduced by Hardy Cross, M. ASCE, offers a very convenient notation for stating these beam properties in a general case. Fig. 1 shows a nonpris-

[2] *Transactions*, ASCE, Vol. 102, 1937, p. 44.

[3] "The Column Analogy," by Hardy Cross, *Bulletin No. 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1932.

matic beam with its load, the corresponding statical moment diagram, the analogous column section, and the indeterminate moment diagram. The diagrams of m_s and m_i have thicknesses normal to the paper equal to the thickness of the analogous column at a corresponding point.

The center of gravity of the analogous column section is indicated in Fig. 1(*c*). This point is considered as the origin of coordinates with x measured positive to the right. The distances to the extreme fibers are shown as c_a and c_b, the former being a negative number. The five fundamental properties are: (*a*) The load P_o on the analogous column, (*b*) its first moment about some reference axis, (*c*) the area of the analogous column A_o, (*d*) its first moment, and (*e*) its second moment.

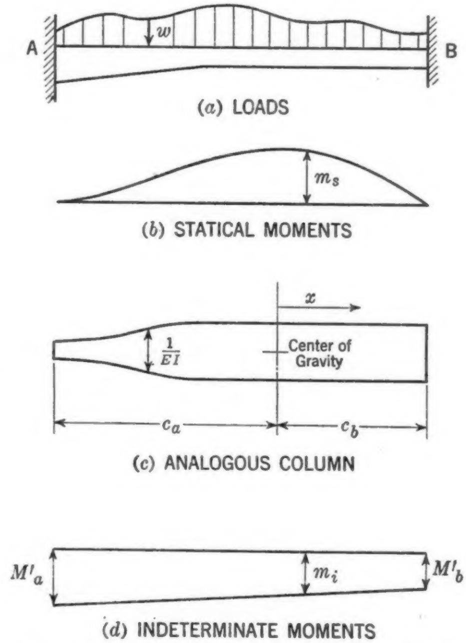


Fig. 1.—Column Analogy for a Single-Span Beam

Corresponding to the fiber distances c_a and c_b, two section moduli S_a and S_b can be defined by

$$S_a = \frac{I_o}{c_a}, \text{ and } S_b = \frac{I_o}{c_b}. \quad (44)$$

in which I_o is the moment of inertia of the column section. The modulus S_a is a negative number since I_o is always positive.

The end moments may be expressed in matrix notation as follows:

$$\begin{bmatrix} M'_a \\ M'_b \end{bmatrix} = \begin{bmatrix} A^{-1}_o & S^{-1}_a \\ A^{-1}_o & S^{-1}_b \end{bmatrix} \begin{bmatrix} P_o \\ M_o \end{bmatrix} \quad \text{.................. (45)}$$

in which M_o is the moment acting on the analogous column, or the moment of P_o about the origin. The column analogy method uses a bending-moment sign convention which defines positive moment as that producing compressive stress in the top fiber. The prime on a moment (M') indicates the fixed-end value of the moment.

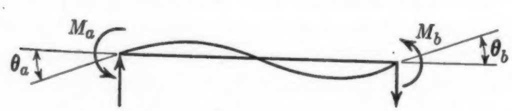


Fig. 2.—Elastic Curve of an Unloaded Span

End Rotation Effect

Next it is necessary to consider the relationship between end moments and end slopes for a single span. Fig. 2 shows an unloaded span with applied end moments. The end moments and end slopes are linearly related. Each end

moment may be expressed as a linear function of the two end slopes, or each end slope may be expressed as a linear function of the two end moments. The former relationship may be written in matrix notation as

$$\begin{bmatrix} M_a \\ M_b \end{bmatrix} = \begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix} \begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

in which K_a and K_b are the standard stiffness values for the member and the quantity R is a secondary stiffness value which gives the change in moment at one end of a member caused by a unit rotation at the opposite end. It is equal to the product of the carry-over factor multiplied by the stiffness, these values being taken at either end. Thus,

$$R = r_a\, K_a = r_b\, K_b \ldots\ldots\ldots\ldots\ldots\ldots\ldots (47)$$

in which r_a and r_b are the carry-over factors. A statical moment sign convention is used in expressing the moment-slope relationship. (The stiffness values and carry-over factors are all positive numbers.) Although a bending moment sign convention may be used in continuous beam analysis, the statical moment convention becomes more convenient when the method of analysis is extended to frame works or trusses. Moment and slope are both chosen to be positive in the counterclockwise direction at either end of the member.

To express the slopes θ_a and θ_b as linear functions of the end moments, multiply Eq. 46 by the reciprocal of the coefficient matrix; thus,

$$\begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix}^{-1} \begin{bmatrix} M_a \\ M_b \end{bmatrix} = \begin{bmatrix} \theta_a \\ \theta_b \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

The reciprocal matrix may be evaluated to give

$$\begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix}^{-1} = \frac{1}{K_a\, K_b - R^2} \begin{bmatrix} K_b & -R \\ -R & K_a \end{bmatrix} \ldots\ldots\ldots\ldots (49)$$

That is,

$$\begin{bmatrix} K_a & R \\ R & K_b \end{bmatrix}^{-1} = \frac{1}{D\, K_a\, K_b} \begin{bmatrix} K_b & -R \\ -R & K_a \end{bmatrix} \ldots\ldots\ldots\ldots (50)$$

in which D is the end rotation constant [4] (commonly indicated by $1/C$) for the beam corresponding to simply-supported end conditions and is given by the formula:

$$D = 1 - r_a\, r_b \ldots\ldots\ldots\ldots\ldots\ldots\ldots (51)$$

The product of the end rotation constant D and the standard stiffness value of a beam gives a modified stiffness value which is the moment required to produce a unit rotation at one end when the other end of the beam is simply supported. When the scalar coefficient of the matrix of Eq. 50 is taken inside the bracket and multiplied by each term, it becomes convenient to introduce new letters to represent the reciprocals of modified stiffness values; thus, let

$$\phi_a = \frac{1}{D\, K_a}\,;\ \phi_b = \frac{1}{D\, K_b}\,;\ \text{and } \gamma = \frac{R}{D\, K_a\, K_b} \ldots\ldots\ldots\ldots (52)$$

[4] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1930, p. 119.

Eq. 48 now becomes

$$\begin{bmatrix}\theta_a\\ \theta_b\end{bmatrix} = \begin{bmatrix}\phi_a & -\gamma\\ -\gamma & \phi_b\end{bmatrix}\begin{bmatrix}M_a\\ M_b\end{bmatrix} \quad \text{(53)}$$

By Eq. 46, each end moment is expressed as a linear function of the two end slopes. By Eq. 53, each end slope is expressed as a linear function of the two end moments.

The numerical values of the stiffness quantities K_a, K_b, and R may be computed by using the column analogy concepts. The value of K_a is the indeterminant moment that would be computed at point A if the analogous column is loaded with a unit concentrated load at point A. The value of R is the indeterminant moment computed at point B with the unit load at point A, and K_b is determined by placing the unit load at point B. Thus, the stiffness values are given by

$$\begin{bmatrix}K_a & R\\ R & K_b\end{bmatrix} = \begin{bmatrix}A^{-1}{}_o & S^{-1}{}_a\\ A^{-1}{}_o & S^{-1}{}_b\end{bmatrix}\begin{bmatrix}1 & 1\\ c_a & c_b\end{bmatrix} \quad \text{(54)}$$

Since the quantities on the right side of Eq. 54 are in accord with a bending moment sign convention, the quantity R as computed will have a negative sign. This sign should be ignored in a statical moment sign convention.

SLOPE-DEFLECTION EQUATIONS FOR SINGLE SPAN

The next step in the analysis requires consideration of a loaded span as shown in Fig. 3. The end moments may be expressed as linear functions of the end slopes as before, but a constant term must be added:

$$\begin{bmatrix}M_a\\ M_b\end{bmatrix} = \begin{bmatrix}K_a & R\\ R & K_b\end{bmatrix}\begin{bmatrix}\theta_a\\ \theta_b\end{bmatrix} + \begin{bmatrix}M'_a\\ M'_b\end{bmatrix} \quad \text{(55)}$$

The elements M'_a and M'_b of the column vector that has been added to Eq. 46 to obtain Eq. 55 represent the values of M_a and M_b when both θ_a and θ_b are zero. Thus, M'_a and M'_b are the fixed-end moments for the given loading and are calculated as shown by Eq. 45. (The values of M'_a and M'_b in Eq. 45 have their signs in accord with a bending moment sign convention and must be changed to a statical moment convention.) Eq. 55 is a special case of the general slope-deflection equations which include an additional term for the effect of relative end, or joint, translation.

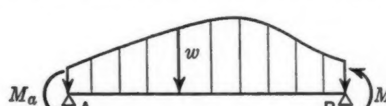


FIG. 3.—LOADED SPAN

It is also possible to use Eq. 53 rather than Eq. 46 to obtain an expression for end slopes in terms of moments for a loaded beam. As before, a constant term is added to obtain

$$\begin{bmatrix}\theta_a\\ \theta_b\end{bmatrix} = \begin{bmatrix}\phi_a & -\gamma\\ -\gamma & \phi_b\end{bmatrix}\begin{bmatrix}M_a\\ M_b\end{bmatrix} + \begin{bmatrix}\theta'_a\\ \theta'_b\end{bmatrix} \quad \text{(56)}$$

The elements θ'_a and θ'_b of the column vector that has been added to Eq. 53 to obtain Eq. 56 represent the values of θ_a and θ_b when both M_a and M_b are

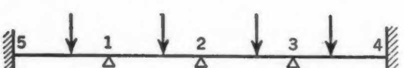
zero. Thus, θ'_a and θ'_b are the simple beam end slopes for the given loading. To evaluate θ'_a and θ'_b, the fixed-end beam is first considered with end moments M'_a and M'_b. If it is now assumed that a moment $-M'_a$ is applied at end A and a moment $-M'_b$ is applied at end B, then the fixed-end beam becomes a simply-supported beam with zero end moments. The angles through which each end rotates, such as $-M'_a$ and $-M'_b$ are applied, will be the angles θ'_a and θ'_b. Therefore, the angles θ'_a and θ'_b are linear functions of $-M'_a$ and $-M'_b$. This relationship is expressed by Eq. 53 as follows:

$$\begin{bmatrix} \theta'_a \\ \theta'_b \end{bmatrix} = - \begin{bmatrix} \phi_a & -\gamma \\ -\gamma & \phi_b \end{bmatrix} \begin{bmatrix} M'_a \\ M'_b \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

This linear relationship between simple beam end slopes and fixed-end moments, for a given loading, has some usefulness in practical design work. It provides a convenient procedure for computing end slopes that are not well known to the designer from stiffness values and fixed-end moments which are well known, either by formula for prismatic beams or by graphical representations for nonprismatic beams.

Slope-Deflection Equations for Entire Beam

A four-span continuous beam is now to be considered as shown in Fig. 4. Beneath each span is shown a 2×2 matrix of stiffness values for the span corresponding to the coefficient matrix of Eq. 46· Since the ends of the beam are fixed, the rotation of joints 4 and 5 is known to be zero. Thus, there are only three effective joints. Only one of the two slope-deflection equations for the end spans and one of the stiffness values for the end spans are needed in determining the bending moments at the effective joints. As a secondary part of the solution, the bending moments at joints 4 and 5 can be computed by proper carry-over calculations according to the process of moment distribution. If the ends of the beam were simply supported rather than fixed, the standard stiffness values K_{15} and K_{34} would be replaced by modified stiffness values using the appropriate end rotation constant. The joints have been numbered by beginning with the first effective joint and continuing consecutively.



$$\begin{bmatrix} * & * \\ * & K_{15} \end{bmatrix} \begin{bmatrix} K_{12} & R_{12} \\ R_{21} & K_{21} \end{bmatrix} \begin{bmatrix} K_{23} & R_{23} \\ R_{32} & K_{32} \end{bmatrix} \begin{bmatrix} K_{34} & * \\ * & * \end{bmatrix}$$

FIG 4.—FOUR-SPAN BEAM, WITH STIFFNESS VALUES

It should be noted that, in dealing with the continuous beam, the necessity arises for using double subscripts on the stiffness values. The first subscript corresponds to the joint, or support, and the second subscript indicates the member. The 2×2 matrices shown in Fig. 4 are symmetrical; and, hence,

$$R_{23} = R_{32}; \text{ and } R_{34} = R_{43} \quad \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

Bending moments will also require double subscripts whereas joint rotations will require only a single subscript.

It is now necessary to define several column vectors. Consider first that the fixed-end moments at the left end of each member form a column vector

$[M'_a]$ and the fixed-end moments at the right end of each member form a column vector $[M'_b]$. Thus,

$$[M'_a] = \begin{bmatrix} M'_{12} \\ M'_{23} \\ M'_{34} \end{bmatrix}; \text{ and } [M'_b] = \begin{bmatrix} M'_{15} \\ M'_{21} \\ M'_{32} \end{bmatrix} \quad \ldots\ldots\ldots\ldots (59)$$

Define similar vectors of the actual moments by

$$[M_a] = \begin{bmatrix} M_{12} \\ M_{23} \\ M_{34} \end{bmatrix}; \text{ and } [M_b] = \begin{bmatrix} M_{15} \\ M_{21} \\ M_{32} \end{bmatrix} \quad \ldots\ldots\ldots\ldots (60)$$

Also define a column vector of joint rotations by

$$[\theta] = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (61)$$

Only the effective joints are included. The vectors $[M_a]$ and $[M_b]$ can be shown to be linear matrix functions of the vector $[\theta]$. Hence,

$$[M_a] = [K_a][\theta] + [M'_a] \quad \ldots\ldots\ldots\ldots (62a)$$

and

$$[M_b] = [K_b][\theta] + [M'_b] \quad \ldots\ldots\ldots\ldots (62b)$$

The similarity in form between Eqs. 62 and Eq. 55 for a single span should be noted. The vector $[M'_a]$ is the value of the vector $[M_a]$ when all elements, or components, of the vector $[\theta]$ are zero. The coefficient matrices $[K_a]$ and $[K_b]$ have been introduced and must be defined. Each is a 3×3 matrix composed of stiffness values properly chosen from Fig. 4. The matrix $[K_a]$ is formed in the following manner: Take the first row of elements of each 2×2 matrix in Fig. 4 and write them one below another—each being staggered one place to the right. Fill the remaining places of the matrix thus formed with zeros. The result is

$$[K_a] = \begin{bmatrix} K_{12} & R_{12} & 0 \\ 0 & K_{23} & R_{23} \\ 0 & 0 & K_{34} \end{bmatrix} \quad \ldots\ldots\ldots\ldots (63a)$$

Using the second row of elements of each 2×2 matrix of Fig. 4, the vector $[K_b]$ is formed as

$$[K_b] = \begin{bmatrix} K_{15} & 0 & 0 \\ R_{21} & K_{21} & 0 \\ 0 & R_{32} & K_{32} \end{bmatrix} \quad \ldots\ldots\ldots\ldots (63b)$$

To demonstrate that Eqs. 62 are true they will be written out in full; thus,

$$\begin{bmatrix} M_{12} \\ M_{23} \\ M_{34} \end{bmatrix} = \begin{bmatrix} K_{12} & R_{12} & 0 \\ 0 & K_{23} & R_{23} \\ 0 & 0 & K_{34} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} M'_{12} \\ M'_{23} \\ M'_{34} \end{bmatrix} \quad \ldots\ldots\ldots\ldots (64a)$$

and

$$\begin{bmatrix} M_{15} \\ M_{21} \\ M_{32} \end{bmatrix} = \begin{bmatrix} K_{15} & 0 & 0 \\ R_{21} & K_{21} & 0 \\ 0 & R_{32} & K_{32} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} M'_{15} \\ M'_{21} \\ M'_{32} \end{bmatrix} \qquad (64b)$$

Each of these two matrix equations corresponds to three algebraic equations. If the multiplication indicated on the right side of the equations is done, a column vector of three components will exist on each side of each equation. Equating corresponding elements of these column vectors will give the two families of three algebraic equations each. Consider the first algebraic equation of the first family as follows:

$$M_{12} = K_{12}\,\theta_1 + R_{12}\,\theta_2 + M'_{12} \qquad (65)$$

Eq. 65 is one of the slope-deflection equations for the second span and corresponds exactly to the first of the two algebraic equations represented by Eq. 55. Similarly all the six algebraic equations corresponding to Eqs. 64 can be shown to be slope-deflection equations for the various spans. Hence, Eqs. 62 are valid.

Eqs. 62 represent a complete statement in matrix notation of the slope-deflection equations for any continuous beam with supports that are fixed against translation. If there are ten effective joints, the column vectors will have ten components and the coefficient matrices will be 10×10 square matrices.

Any numerical values for the components of $[\theta]$—that is, the joint rotations—may be assumed and substituted into Eqs. 62. The resulting moments will correspond to a continuous elastic curve for the beam. The moment on the left side of each joint is not equal in magnitude to the moment on the right side of the joint. From a physical standpoint this state of deflections and moments could exist only if artificial moments of sufficient magnitude to maintain the joint rotations at the assumed values were introduced at the joints. These artificial moments might be introduced by considering that torques are applied to axles which are fixed to the beam at the joints and are normal to the plane of the page.

Actually, the foregoing solution is not correct for moments resulting from the applied loads. There is one, and only one, correct set of values of the components of $[\theta]$. With this correct set of values the moment on the left side of each support is equal to the moment on the right side of the same support, but of opposite sign. In other words, each joint must be in equilibrium so that no artificial torques are required. Thus, the correct solution must satisfy both the continuity of the elastic curve and the equilibrium of the joints. Eqs. 62 guarantee continuity but do not guarantee equilibrium. Hence, the equilibrium conditions must also be imposed on the solution.

If each joint is in equilibrium, the elements of the vector $[M_a]$ must be equal and opposite in sign to the elements of $[M_b]$. Thus, the equilibrium condition is expressed by

$$[M_a] + [M_b] = 0 \qquad (66)$$

Eqs. 62 and 66 define the solution completely. The vector $[\theta]$ can be elim-

inated from these three matrix equations to give convenient formulas for design practice.

THEOREM OF THREE SLOPES

If Eqs. 62 are substituted into Eq. 66,

$$[K][\theta] + [u] = 0 \quad \ldots\ldots (67)$$

in which

$$[K] = [K_a] + [K_b] = \begin{bmatrix} \Sigma_1 K & R_{12} & 0 \\ R_{21} & \Sigma_2 K & R_{23} \\ 0 & R_{32} & \Sigma_3 K \end{bmatrix} \quad \ldots\ldots (68)$$

and

$$[u] = [M'_a] + [M'_b] = \begin{bmatrix} M'_{12} + M'_{15} \\ M'_{23} + M'_{21} \\ M'_{34} + M'_{32} \end{bmatrix} \quad \ldots\ldots (69)$$

The elements of the stiffness matrix $[K]$ should be considered. The first element of the principal diagonal is the sum of stiffness values at joint 1. The second element of the principal diagonal is the sum of stiffness values at joint 2, etc. The matrix contains one diagonal row of elements above and one below the principal diagonal, and is symmetrical. These properties are true for the stiffness matrix of all continuous beams regardless of the number of effective joints. The diagonal row above the principal diagonal is called the "superdiagonal" and the diagonal row below the principal diagonal is called the "subdiagonal." A matrix that has a principal diagonal, subdiagonal, and superdiagonal, with all other elements equal to zero, is called a "continuant matrix." [5] (The mathematical reason for this name has been stated by A. C. Aitken.) The stiffness matrix $[K]$ is a continuant matrix. This will only be true if the effective joints are numbered in consecutive order. The subdiagonal and superdiagonal are composed of secondary stiffness values, or R-values. In any given row the diagonal element corresponds to a particular joint. The R-values in this row correspond to the members adjacent to the joint.

The vector $[u]$ is composed of components that may be recognized as the "unbalanced" moments of the moment distribution process of analysis. The unbalanced moment at a joint is the sum of the fixed-end moments at the joint. The vector $[u]$ may be written

$$[u] = \begin{bmatrix} \Sigma_1 M' \\ \Sigma_2 M' \\ \Sigma_3 M' \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad \ldots\ldots (70)$$

For example, from Eq. 69, $u_2 = M'_{23} + M'_{21}$.

Each of the algebraic equations represented by the matrix Eq. 67 contains three slopes, or joint rotations, irrespective of the number of spans of the continuous beam. Hence, Eq. 67 may be called the theorem of three slopes (or "angles," which is a term used by W. L. Schwalbe[6]). This equation can be

[5] "Matrices and Determinants," by A. C. Aitken, Interscience Publishers, Inc., New York, N. Y., 1939, p. 126.

[6] "Simultaneous Equations in Mechanics Solved by Iteration," by W. L. Schwalbe, *Transactions* ASCE, Vol. 102, 1937, p. 941.

solved for $[\theta]$ by multiplying the equation by $[K]^{-1}$:

$$[\theta] = -[K]^{-1}[u] \qquad (71)$$

Eq. 71 states that the joint rotations created by a given loading are linear functions of the unbalanced moments. Substituting Eq. 71 in Eqs. 62 gives

$$[M_a] = [M'_a] - [K_a][K]^{-1}[u] \qquad (72a)$$

and

$$[M_b] = [M'_b] - [K_b][K]^{-1}[u] \qquad (72b)$$

The second term on the right-hand side of Eqs. 72 may be considered a correction or modification to be added, with its negative sign, to the fixed-end moment. The final moment at a joint may be considered to be composed of a preliminary estimate, or fundamental value—the fixed-end moment plus a correction that is a linear function of the unbalanced moments of all the joints.

It is convenient to introduce the following matrices:

$$[C_a] = -[K_a][K]^{-1} \qquad (73a)$$

and

$$[C_b] = -[K_b][K]^{-1} \qquad (73b)$$

Eqs. 72 become

$$[M_a] = [M'_a] + [C_a][u] \qquad (74a)$$

and

$$[M_b] = [M'_b] + [C_b][u] \qquad (74b)$$

The numerical values of joint moments may be computed by using either Eq. 74*a* or Eq. 74*b*. However, it is advisable to use both and to compare the values as this procedure checks the computation of the elements of $[C_a]$ and $[C_b]$. Eqs. 74 give a complete statement in matrix notation of the solution of any continuous beam of any number of spans. The column vectors $[M'_a]$, $[M'_b]$, and $[u]$ are dependent on the loading and physical characteristics of the individual spans. The coefficient matrices $[C_a]$ and $[C_b]$ are dependent only on the physical properties of the structure. They can be computed for a given structure before the loading is known and need to be computed only once. The information necessary for this computation is shown in Fig. 4.

The matrix $[C_a]$ is computed in two steps: First, $[K]^{-1}$ is computed; and then this value is premultiplied by $[K_a]$. A matrix multiplication can be performed in a few minutes, particularly on modern calculators which retain and add products of numbers. Large matrices can be multiplied faster by using a calculating machine than by using a slide rule since the result of each individual multiplication does not have to be written down but is retained in the machine. The principal labor involved is in the computation of $[K]^{-1}$. The time required for computating the reciprocal of a matrix increases very rapidly with the size of the matrix. For example, the calculation of the reciprocal of a 6×6 matrix would require the evaluation of thirty-six 5×5 determinants—a very lengthy process.

It is convenient, for discussion, to introduce the correction vectors $[a]$ and $[b]$ by the definitions:

$$[a] = [C_a][u]; \text{ and } [b] = [C_b][u] \qquad (75)$$

Eqs. 74 become

$$[M_a] = [M'_a] + [a] \qquad (76a)$$

and

$$[M_b] = [M'_b] + [b] \qquad (76b)$$

The design computations for a given loading can be written in a manner similar to moment distribution after the elements of the coefficient matrices $[C_a]$ and $[C_b]$ have been computed. The fixed-end moments may be written, as usual, and directly below them the components of the correction vectors $[a]$ and $[b]$ are written. Direct addition at each joint gives the final answer. Because of the length of time required to compute the reciprocal of the stiffness matrix $[K]$, and thus to obtain $[C_a]$ and $[C_b]$, a given beam can be analyzed faster for one loading condition by moment distribution than by using matrices. The advantage of using matrices would become apparent if a beam were to be analyzed for many loading conditions, since the reciprocal of $[K]$ would need to be computed only once—suggesting that matrix methods should prove to be useful in developing influence lines.

Theorem of Three Moments

The theorem of three moments is better known to structural engineers than the theorem of three slopes. In developing the theorem of three moments, it is necessary to begin with the moment-slope relationship of Eq. 56. The end slopes are considered linear functions of the end moments. Proceeding in a manner similar to the development of the theorem of three slopes, a complete set of moment-slope relationships for the entire continuous beam can be written. This set of equations will guarantee equilibrium rather than continuity. The condition of continuity at each joint must be introduced and the theorem of three moments is then obtained. It expresses the joint moments as functions of the simple beam end slopes and can be solved directly for the moments. Eq. 57 has been given for obtaining simple beam end slopes from fixed-end moments.

To develop the theorem of three moments, it is necessary to shift to a bending moment sign convention. It is also necessary to differentiate between the slopes (rather than the moments) on the right-hand side and left-hand side of a support. Thus, slopes must have double subscripts whereas joint moments need have only single subscripts. Since these changes are very confusing, the development has been omitted, the outline merely being given to indicate a possibility for those who may wish to pursue the study further.

Numerical Example

In Fig. 5, a continuous beam is shown with parabolic haunches and simple supports. A row of standard stiffness values for each span is shown as deter-

mined from graphs published by the Portland Cement Association.[7] Since these graphs give the carry-over factors, the secondary stiffness values must be computed. Just below the stiffness values are the stiffness matrices for each span, showing a modified stiffness for the end spans caused by the simple

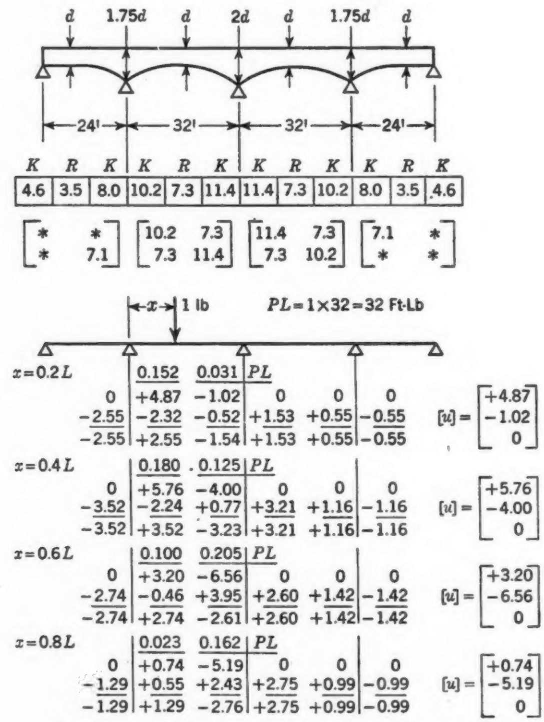


FIG. 5.—ANALYSIS OF A FOUR-SPAN CONTINUOUS BEAM

supports at the ends. The left-end and right-end stiffness matrices $[K_a]$ and $[K_b]$ are

$$[K_a] = \begin{bmatrix} 10.2 & 7.3 & 0 \\ 0 & 11.4 & 7.3 \\ 0 & 0 & 7.1 \end{bmatrix} \quad \text{(77a)}$$

and

$$[K_b] = \begin{bmatrix} 7.1 & 0 & 0 \\ 7.3 & 11.4 & 0 \\ 0 & 7.3 & 10.2 \end{bmatrix} \quad \text{(77b)}$$

By adding Eqs. 77*a* and 77*b*, the stiffness matrix $[K]$ is obtained

$$[K] = \begin{bmatrix} 17.3 & 7.3 & 0 \\ 7.3 & 22.8 & 7.3 \\ 0 & 7.3 & 17.3 \end{bmatrix} \quad \text{(78)}$$

[7] "Continuous Concrete Bridges," Portland Cement Assn., Chicago, Ill., 1939.

The reciprocal is given by

$$[K]^{-1} = \frac{1}{4{,}980}\begin{bmatrix} 341 & -126 & 53.3 \\ -126 & 299 & -126 \\ 53.3 & -126 & 341 \end{bmatrix} = \begin{bmatrix} 0.0685 & -0.0253 & 0.0107 \\ -0.0253 & 0.0600 & -0.0253 \\ 0.0107 & -0.0253 & 0.0685 \end{bmatrix} \quad \text{..(79)}$$

The matrices $[K]$ and $[K]^{-1}$ are symmetrical about the principal diagonal and also about the diagonal running from the upper right-hand corner downward (the secondary diagonal). This second symmetry occurs because the beam is itself symmetrical about its center support.

The matrices $[C_a]$ and $[C_b]$ are computed to be

$$[C_a] = -[K_a][K]^{-1} = \begin{bmatrix} -0.514 & -0.180 & +0.076 \\ +0.210 & -0.499 & -0.212 \\ -0.076 & +0.180 & -0.486 \end{bmatrix} \quad \text{......(80}a\text{)}$$

and

$$[C_b] = -[K_b][K]^{-1} = \begin{bmatrix} -0.486 & +0.180 & -0.076 \\ -0.212 & -0.499 & +0.210 \\ +0.076 & -0.180 & -0.514 \end{bmatrix} \quad \text{......(80}b\text{)}$$

It should be noted that $[C_b]$ can be obtained from $[C_a]$ by two rotations of the matrix—one about the principal diagonal and one about the secondary diagonal. This condition is caused by symmetry of the beam about its center support.

The correction vectors $[a]$ and $[b]$ can now be written as

$$[a] = \begin{bmatrix} -0.514 & -0.180 & +0.076 \\ +0.210 & -0.499 & -0.212 \\ -0.076 & +0.180 & -0.486 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad \text{............(81}a\text{)}$$

and

$$[b] = \begin{bmatrix} -0.486 & +0.180 & -0.076 \\ -0.212 & -0.499 & +0.210 \\ +0.076 & -0.180 & -0.514 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad \text{............(81}b\text{)}$$

For any given loading condition, fixed-end moments can be computed from graphical representations of moment coefficients. The unbalanced moments $[u]$ can then be determined, and the corrections $[a]$ and $[b]$ can be computed by a matrix multiplication as shown in Eqs. 81. The convenience of using this method for calculating influence lines should be apparent.

Four solutions for four positions of a unit load in the second span are shown in Fig. 5. The components of the correction vectors have been computed in each case by a matrix multiplication using Eqs. 81. The fixed-end moment coefficients have been taken from a booklet issued by the Portland Cement Association.[7] The column vector of unbalanced moments is shown at the right for each case.

PART III. MATRIX DEVELOPMENT OF MOMENT DISTRIBUTION

Introduction

This development is also limited to the consideration of continuous beams on supports which are fixed against translation. The process of moment dis-

tribution consists of a sequence of cycles of numerical computations. Each cycle has two steps. In the first step all joints are balanced and in the second step the "carry-over calculation" is performed in each span. Although other sequences of performing the operations are possible, the foregoing order has generally been adopted because of the ease in checking the computations by an individual other than the original computer. A statical moment sign convention will be used through this part of the paper. (After submission of the manuscript for publication a discussion by Leon Beskin, Assoc. M. ASCE, appeared in which several matrix equations were presented corresponding closely to some of those contained in this part of the paper.[8])

Beam with One Effective Joint

It is possible to give a simple illustration of the process of moment distribution by considering a beam for which the final moments can be obtained by performing only one cycle of computations. Such a beam is one having only one effective joint. Three examples are shown in Fig. 6. These beams do not

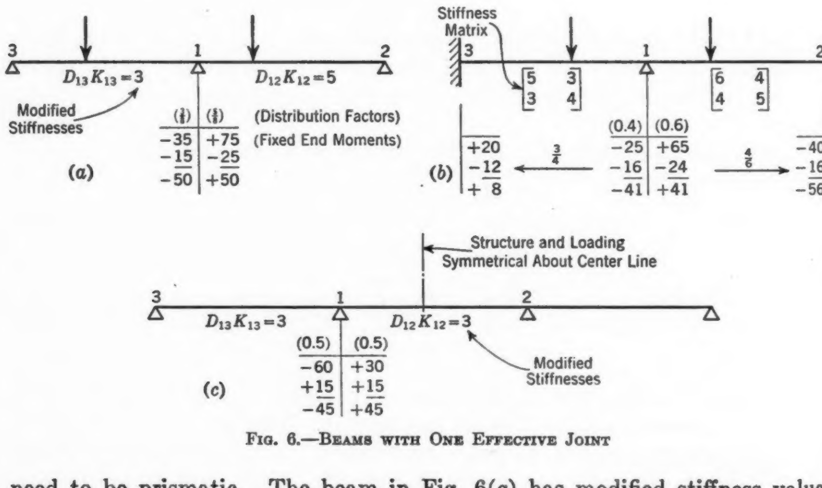


Fig. 6.—Beams with One Effective Joint

need to be prismatic. The beam in Fig. 6(*a*) has modified stiffness values, computed by using the appropriate end rotation constants. It is only necessary to perform the first step of the first cycle of moment distribution to obtain the final solution. A similar type of case is shown in Fig. 6(*c*) in which a single distribution of moments at joint 1 gives the final solution. The opportunity for using an end rotation constant in the center span arises from the symmetry of the structure and its loading. A similar opportunity would exist if the loading were antisymmetrical. In Fig. 6(*b*) is shown a beam that requires both steps of the cycle for its complete solution. The stiffness matrix for each span is shown, from which the distribution and carry-over factors are computed. The distribution factors are shown in parentheses above the moment distribution. The distribution step of the cycle yields the correction values which,

[8] *Proceedings*, ASCE, September, 1945, pp. 1111–1120

when added to the fixed-end moments, give the final moments at joint 1. The carry-over step determines the outer end moments.

In each of the examples of Fig. 6, the computations at the effective joint conform to the following algebraic equations:

$$M_{12} = M'_{12} + a_1 \ldots\ldots\ldots\ldots\ldots\ldots (82a)$$

and

$$M_{13} = M'_{13} + b_1 \ldots\ldots\ldots\ldots\ldots\ldots (82b)$$

in which

$$a_1 = -\frac{K_{12}}{\Sigma_1 K} u_1 = -d_{12}\,u_1 \ldots\ldots\ldots\ldots\ldots (83a)$$

and

$$b_1 = -\frac{K_{13}}{\Sigma_1 K} u_1 = -d_{13}\,u_1 \ldots\ldots\ldots\ldots\ldots (83b)$$

The final moments are obtained by adding to the fixed-end moments the corrections a and b. This procedure corresponds exactly to the method of solution described in Part II and illustrated by Eqs. 76. In the present case the column vectors have only one element and, hence, are ordinary numbers. The distribution factors d_{12} and d_{13} have been introduced and defined in agreement with the usual moment distribution procedure.

Matrix Reciprocal Expressed by Power Series

As was stated in Part I, most of the work involved in solving a continuous beam of many spans lies in the evaluation of the reciprocal of the stiffness matrix $[K]$. Instead of calculating this reciprocal by using determinants as explained in Part I, it is possible to obtain this reciprocal by a converging approximation method. The method can be developed by expressing $[K]^{-1}$ as a power series of matrices. This process leads directly to a complete mathematical explanation of moment distribution.

The matrix $[K]$, for a beam with three effective joints (see Fig. 4), is given by

$$[K] = \begin{bmatrix} \Sigma_1 K & R_{12} & 0 \\ R_{21} & \Sigma_2 K & R_{23} \\ 0 & R_{32} & \Sigma_3 K \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (84)$$

This matrix can be separated into two parts defined as follows:

$$[D] = \begin{bmatrix} \Sigma_1 K & 0 & 0 \\ 0 & \Sigma_2 K & 0 \\ 0 & 0 & \Sigma_3 K \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (85)$$

and

$$[R] = \begin{bmatrix} 0 & -R_{12} & 0 \\ -R_{21} & 0 & -R_{23} \\ 0 & -R_{32} & 0 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots (86)$$

The matrix $[D]$ is a diagonal matrix containing the diagonal elements of $[K]$. The matrix $[R]$ contains the nondiagonal elements of $[K]$ with negative signs. The reciprocal of $[D]$ will appear in the development and a convenient rule

for computing the reciprocal of a diagonal matrix can be established. Evaluating $[D]^{-1}$ by determinants gives

$$[D]^{-1} = \frac{1}{\Sigma_1 K\,\Sigma_2 K\,\Sigma_3 K}\begin{bmatrix} \Sigma_2 K\,\Sigma_3 K & 0 & 0 \\ 0 & \Sigma_1 K\,\Sigma_3 K & 0 \\ 0 & 0 & \Sigma_2 K\,\Sigma_3 K \end{bmatrix}$$

$$= \begin{bmatrix} (\Sigma_1 K)^{-1} & 0 & 0 \\ 0 & (\Sigma_2 K)^{-1} & 0 \\ 0 & 0 & (\Sigma_3 K)^{-1} \end{bmatrix} \qquad (87)$$

Inspection shows that $[D]^{-1}$ is also a diagonal matrix formed by replacing each of the diagonal elements of $[D]$ by its reciprocal.

From Eqs. 84 to 86:

$$[K] = [D] - [R] \qquad (88)$$

Eq. 88 may be written as

$$[K] = \{[I] - [R][D]^{-1}\}[D] \qquad (89)$$

It becomes convenient to define the matrix $[Q]'$ by

$$[Q]' = [R][D]^{-1} \qquad (90)$$

The physical significance of the elements of $[Q]'$ and the reason for using the transpose $[Q]'$ instead of $[Q]$ will be explained subsequently. Eq. 89 becomes

$$[K] = \{[I] - [Q]'\}[D] \qquad (91)$$

To write the reciprocal of $[K]$, it is necessary to establish another rule of matrix algebra regarding the reciprocal of a product of two matrices. For example, assume that

$$[C] = [A][B] \qquad (92)$$

Premultiplying both sides of Eq. 92 by $[A]^{-1}$,

$$[A]^{-1}[C] = [I][B] = [B] \qquad (93)$$

Premultiplying both sides of Eq. 93 by $[B]^{-1}$,

$$[B]^{-1}[A]^{-1}[C] = [I] \qquad (94)$$

Postmultiplying both sides of Eq. 94 by $[C]^{-1}$,

$$[B]^{-1}[A]^{-1} = [C]^{-1} \qquad (95)$$

Comparing Eq. 95 with Eq. 92, it is apparent that the reciprocal of a product of two matrices equals the product of their reciprocals—in reverse order. From this rule, and from Eq. 91, $[K]^{-1}$ can be written

$$[K]^{-1} = [D]^{-1}\{[I] - [Q]'\}^{-1} = [D]^{-1}\left\{\frac{[I]}{[I] - [Q]'}\right\} \qquad (96)$$

In general, it is not permissible to write a matrix division as a fraction because of the ambiguity that arises as explained in Part I. However, the fraction shown in Eq. 96 has the identity matrix for a numerator; and, hence, no

ambiguity arises. The reciprocal has been written in the form of a fraction in order that it can be compared with the following well-known algebraic formula:

$$\frac{1}{1-\epsilon} = 1 + \epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4 + \cdots \qquad (97)$$

This infinite power series can be derived by applying the binomial theorem or by direct long division. The number unity of ordinary numbers is comparable to the identity matrix in matrix algebra. Hence, by comparison:

$$\{[I] - [Q]'\}^{-1} = [I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots \qquad (98)$$

The validity of Eq. 98 can be demonstrated by multiplying both sides by the binomial $[I] - [Q]'$. A rigorous mathematical proof of the correctness of this series requires that the power matrix $\{[Q]'\}^n$ shall approach the zero matrix as n approaches infinity. The reciprocal $[K]^{-1}$ can now be expressed by

$$[K]^{-1} = [D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \qquad (99)$$

Moment Distribution

As was shown in Part II, the correction vectors $[a]$ and $[b]$ are given by the following formulas:

$$[a] = -[K_a][K]^{-1}[u] \qquad (100a)$$

and

$$[b] = -[K_b][K]^{-1}[u] \qquad (100b)$$

Substituting Eq. 99 into Eqs. 100,

$$[a] = -[K_a][D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \{[Q]'\}^4 + \cdots\}[u] \qquad (101a)$$

and

$$[b] = -[K_b][D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \{[Q]'\}^4 + \cdots\}[u] \qquad (101b)$$

The physical significance of these equations can be noted by writing the matrices in expanded form and considering the operations term by term. First, consider the two matrices in front of the heavy brace in each equation. Assume a beam having three effective joints:

$$[K_a][D]^{-1} = \begin{bmatrix} K_{12} & R_{12} & 0 \\ 0 & K_{23} & R_{23} \\ 0 & 0 & K_{34} \end{bmatrix} \begin{bmatrix} (\Sigma_1 K)^{-1} & 0 & 0 \\ 0 & (\Sigma_2 K)^{-1} & 0 \\ 0 & 0 & (\Sigma_3 K)^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} d_{12} & r_{21}\, d_{21} & 0 \\ 0 & d_{23} & r_{32}\, d_{32} \\ 0 & 0 & d_{34} \end{bmatrix} \qquad (102)$$

In this matrix the diagonal elements are the distribution factors applicable at the left end of the members (or the right side of the joints) as used in moment distribution. The nondiagonal elements are products of a distribution factor

and a corresponding carry-over factor. It is convenient to represent these nondiagonal elements by a single letter q with appropriate subscripts. Thus, for example,

$$q_{21} = r_{21}\, d_{21}; \text{ and } q_{32} = r_{32}\, d_{32} \qquad (103)$$

It is appropriate to discuss the relationship of the quantity q to other physical quantities which are well known. If the distribution factor d_{23}, as used in moment distribution, is referred to as a primary distribution factor, then the quantity q_{23} may be called a secondary distribution factor. This is analogous to calling K_{23} the primary stiffness value and R_{23} the secondary stiffness value as introduced in Part II. In both cases the secondary value is obtained by multiplying the primary value by the carry-over factor r_{23}. Thus,

$$R_{23} = r_{23}\, K_{23}; \text{ and } q_{23} = r_{23}\, d_{23} \qquad (104)$$

It is also possible to compute the secondary distribution factor in a manner which is exactly the same as that used for the primary factors. Thus,

$$d_{23} = \frac{K_{23}}{\Sigma_2 K}\ ; \text{ and } q_{23} = \frac{R_{23}}{\Sigma_2 K} \qquad (105)$$

Since the distribution factors can be calculated directly from the corresponding stiffness values, an analysis of a continuous beam by matrix methods does not require computation of the carry-over factor. Introducing secondary distribution factors, Eq. 102 becomes

$$[K_a][D]^{-1} = \begin{bmatrix} d_{12} & q_{21} & 0 \\ 0 & d_{23} & q_{32} \\ 0 & 0 & d_{34} \end{bmatrix} \qquad (106a)$$

and, similarly,

$$[K_b][D]^{-1} = \begin{bmatrix} d_{15} & 0 & 0 \\ q_{12} & d_{21} & 0 \\ 0 & q_{23} & d_{32} \end{bmatrix} \qquad (106b)$$

Now consider the terms of the power series of Eqs. 101, term by term. First, assume that the correction vectors are to be computed using only the first term of the series:

$$[a] = -\,[K_a][D]^{-1}\,[I][u] = -\,[K_a][D]^{-1}\,[u] \qquad (107a)$$

and

$$[b] = -\,[K_b][D]^{-1}\,[I][u] = -\,[K_b][D]^{-1}\,[u] \qquad (107b)$$

Substituting Eqs. 106,

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = - \begin{bmatrix} d_{12} & q_{21} & 0 \\ 0 & d_{23} & q_{32} \\ 0 & 0 & d_{34} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (108a)$$

and

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = - \begin{bmatrix} d_{15} & 0 & 0 \\ q_{12} & d_{21} & 0 \\ 0 & q_{23} & d_{32} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (108b)$$

When the first element of the first row of each coefficient matrix is multiplied by the first element of the vector [u], the two quantities obtained are $-d_{12}\,u_1$ and $-d_{15}\,u_1$—the products of the unbalanced moment at joint 1 and the distribution factors. The contribution $-d_{12}\,u_1$ to the value of a_1 is thus exactly the value of the contribution to the final joint moment which is made by the first step of a moment distribution cycle applied to the unbalanced moment at joint 1. Similarly, the products of all the diagonal elements of the coefficient matrices of Eqs. 108 and the vector [u] provide the same contributions to the solution as are provided by the first step of the first cycle of moment distribution in which all the joints are balanced.

Consider next the effect of the nondiagonal elements, or secondary distribution factors, of the coefficient matrices. Consider only the elements of the second row of the matrix of Eq. 108*a*. The nondiagonal element q_{32} contributes $-q_{32}\,u_3$ to the value of a_2. From the definition of q, the contribution is $-r_{32}\,d_{32}\,u_3$. This quantity is the contribution to the joint moment on the right side of joint 2 brought about by the carry-over calculation in span 23. Thus, the nondiagonal elements of the coefficient matrices correspond to the second step of a moment distribution cycle.

Consequently, the coefficient matrices of Eqs. 108 represent, in matrix notation, one complete cycle of moment distribution. These matrices—$-[K_a][D]^{-1}$ and $-[K_b][D]^{-1}$—when applied to a column vector [u] of unbalanced moments make the same contribution to the final solution as is made by one complete cycle of moment distribution.

Consider now that the correction vectors [a] and [b] are to be computed by using the first two terms of the series of Eqs. 101:

$$[a] = -[K_a][D]^{-1}[u] - [K_a][D]^{-1}[Q]'[u] \qquad \text{(109a)}$$

and

$$[b] = -[K_b][D]^{-1}[u] - [K_b][D]^{-1}[Q]'[u] \qquad \text{(109b)}$$

The second terms could be expressed in a manner similar to Eqs. 108, given for the first term, if the vector [u] were replaced by the product $[Q]'[u]$. In expanded form, this product (see Eq. 90) becomes

$$[Q]'[u] = [R][D]^{-1}[u]$$

$$= \begin{bmatrix} 0 & -R_{12} & 0 \\ -R_{21} & 0 & -R_{23} \\ 0 & -R_{32} & 0 \end{bmatrix} \begin{bmatrix} (\Sigma_1 K)^{-1} & 0 & 0 \\ 0 & (\Sigma_2 K)^{-1} & 0 \\ 0 & 0 & (\Sigma_3 K)^{-1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad \text{(110)}$$

Let

$$[u'] = [Q]'[u] \qquad \text{(111)}$$

Then

$$[u'] = -\begin{bmatrix} 0 & r_{21}\,d_{21} & 0 \\ r_{12}\,d_{12} & 0 & r_{32}\,d_{32} \\ 0 & r_{23}\,d_{23} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = -\begin{bmatrix} 0 & q_{21} & 0 \\ q_{12} & 0 & q_{32} \\ 0 & q_{23} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad \text{(112)}$$

Consideration of the coefficient matrix $[Q]'$ of Eq. 112 shows why the transpose $[Q]'$ was introduced rather than [Q]. The subscripts of the elements

show that the matrix is in transposed form. Calculating the components of $[u']$ individually gives

$$u'_1 = - r_{21}\, d_{21}\, u_2 \ldots\ldots (113a)$$

$$u'_2 = - r_{12}\, d_{12}\, u_1 - r_{32}\, d_{32}\, u_3 \ldots\ldots (113b)$$

and

$$u'_3 = - r_{23}\, d_{23}\, u_2 \ldots\ldots (113c)$$

From consideration of these formulas, the components of $[u']$ are shown to be the unbalanced moments remaining at the joints after one cycle of moment distribution has been completed. Since it was previously shown that the coefficients $-[K_a][D]^{-1}$ and $-[K_b][D]^{-1}$ correspond to one cycle of moment distribution, it should now be apparent that the first term of Eqs. 109 corresponds to the first cycle of moment distribution and the second term corresponds to the second cycle.

Similarly, it can be shown that the third term of the power series of Eqs. 101 corresponds to the third cycle of moment distribution, etc. Thus, it is established that a complete mathematical statement of the method of moment distribution can be given in matrix notation as follows:

$$[M_a] = [M'_a] - [K_a][D]^{-1} \{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\}[u] \ldots (114a)$$

and

$$[M_b] = [M'_b] - [K_b][D]^{-1} \{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\}[u] \ldots (114b)$$

Although, by making certain limiting assumptions, R. Oldenburger proved the convergence of moment distribution, this theory has not been given very extensive treatment by mathematicians.[9] Eqs. 114 show that this convergence is dependent upon the convergence of the series within the braces. In any given practical case, a rigorous proof of convergence can be developed; but it will not be attempted here.

PART IV. VARIATIONS FROM MOMENT DISTRIBUTION

Introduction

In Part II, it was shown that a complete solution can be obtained by matrix computation methods in which the reciprocal of the stiffness matrix is computed by using determinants. With Eqs. 114, it is possible to devise a number of different converging approximation methods of solution using matrix methods of computation. The method of moment distribution, with its cycles of numerical computations, is in exact correspondence with Eqs. 114; yet it makes no use of matrix methods. It is possible to devise combined procedures which use both matrix methods and moment distribution. A development of two methods, based upon Eqs. 114, will be given.

[9] "Convergence of Hardy Cross's Balancing Process," by R. Oldenburger, *Journal of Applied Mechanics*, December, 1940, pp. A-166 to A-170.

A Combined Method

If a given beam is to be analyzed for only one loading condition, the designer who is familiar with moment distribution will undoubtedly find it the fastest method of solution available. However, if it is desired to use matrix methods, an interesting combined procedure can be developed by writing the correction vectors $[a]$ and $[b]$ as follows:

$$[a] = -[K_a][D]^{-1}[\bar{u}] \qquad (115a)$$

and

$$[b] = -[K_b][D]^{-1}[\bar{u}] \qquad (115b)$$

in which $[\bar{u}]$ is a vector of transformed unbalanced moments defined by

$$[\bar{u}] = [u] + [u'] + [u''] + [u'''] + \cdots \qquad (116)$$

The vectors $[u']$, $[u'']$, etc., are the unbalanced moments which remain after each cycle of moment distribution and may be computed by the formulas:

$$[u'] = [Q]'[u] \qquad (117a)$$

$$[u''] = \{[Q]'\}^2 [u] = [Q]'[u'] \qquad (117b)$$

$$[u'''] = \{[Q]'\}^3 [u] = [Q]'[u''] \qquad (117c)$$

Each of these vectors is computed by multiplying the previous one by $[Q]'$. It is interesting, and rather surprising, to discover that the unbalanced moments which will exist after any given number of cycles of moment distribution can be computed directly without actually writing out the moment distribution computations. The elements of the vectors $[u']$, $[u'']$, etc., decrease and rapidly approach zero. To obtain the accuracy, to three significant places, which is customary in structural design practice, in general, the four vectors of the type given by Eqs. 117 will be sufficient. Each additional vector usually gives one additional significant figure accurately. The accuracy of the final solution, however, cannot be greater than the accuracy of the fixed-end moments, which are seldom obtainable to an accuracy greater than three significant places. For several practical reasons, significant figures in excess of three are generally found of little use in design practice.

After the transformed vector $[\bar{u}]$ has been computed, the correction vectors $[a]$ and $[b]$ can be determined from Eqs. 115. However, it has been shown previously that the contributions to a solution made by premultiplying a vector of unbalanced moments by the factors $-[K_a][D]^{-1}$ and $-[K_b][D]^{-1}$ correspond to operating on the vector with one cycle of moment distribution. Thus, it becomes apparent that a solution can be obtained by first computing $[\bar{u}]$ and then performing one cycle of moment distribution with this set of unbalanced moments rather than with the actual values of $[u]$. If any joint is slightly out of balance at the end of the cycle, such a joint should be balanced. The foregoing procedure employs both matrix methods and moment distribution. It is recommended only for beams that are to be analyzed for one or two loading conditions or, more specifically, when the number of loading conditions is less than the number of effective joints.

A Matrix Method

If a continuous beam is to be analyzed for a number of loading conditions, equal to or greater than the number of effective joints, a different process of analysis will be found to require less time than the one previously described. This second method employs only matrix methods and becomes especially useful when the number of loading conditions greatly exceeds the number of effective joints. One such case is evidently the computation of influence lines by placing a unit load at various points along the beam.

Referring to Eqs. 101, the complete coefficients of $[u]$ may be evaluated to obtain the coefficient matrices $[C_a]$ and $[C_b]$ as introduced in Part II (see Eq. 75). The vectors $[a]$ and $[b]$ become

$$[a] = [C_a][u]; \text{ and } [b] = [C_b][u] \quad \text{(118)}$$

From Eqs. 101, the formulas for $[C_a]$ and $[C_b]$ become

$$[C_a] = -[K_a][D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \quad \text{(119a)}$$

and

$$[C_b] = -[K_b][D]^{-1}\{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \quad \text{(119b)}$$

Eqs. 118 and 119 provide a method of successive corrections for calculating $[C_a]$ and $[C_b]$ that may be used instead of determinants as explained in Part II.

The method of successive corrections will be found to require less time than the method of determinants when the beam has four or more effective joints—that is, when $[Q]$ is of the fourth or higher order. Substituting Eqs. 106 into Eqs. 119,

$$[C_a] = -\begin{bmatrix} d_{12} & q_{21} & 0 \\ 0 & d_{23} & q_{32} \\ 0 & 0 & d_{34} \end{bmatrix} \{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \quad \text{(120a)}$$

and

$$[C_b] = -\begin{bmatrix} d_{15} & 0 & 0 \\ q_{12} & d_{21} & 0 \\ 0 & q_{23} & d_{32} \end{bmatrix} \{[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \cdots\} \quad \text{(120b)}$$

The first step in the numerical computations is the evaluation of the matrix obtained by summing the power series within the large braces. Usually the fourth-degree term in the series will be the highest power of $[Q]'$ which is needed. Each term of the series is computed by multiplying the previous term by $[Q]'$. In order to perform this calculation, it is necessary for the designer to have a convenient rule for setting up the matrix $[Q]'$ in expanded form. Referring to Eq. 112, for a beam with three effective joints, $[Q]'$ has the form:

$$[Q]' = -\begin{bmatrix} 0 & q_{21} & 0 \\ q_{12} & 0 & q_{32} \\ 0 & q_{23} & 0 \end{bmatrix} \quad \text{(121)}$$

This matrix consists solely of secondary distribution factors. Considering the matrix $[Q]$,

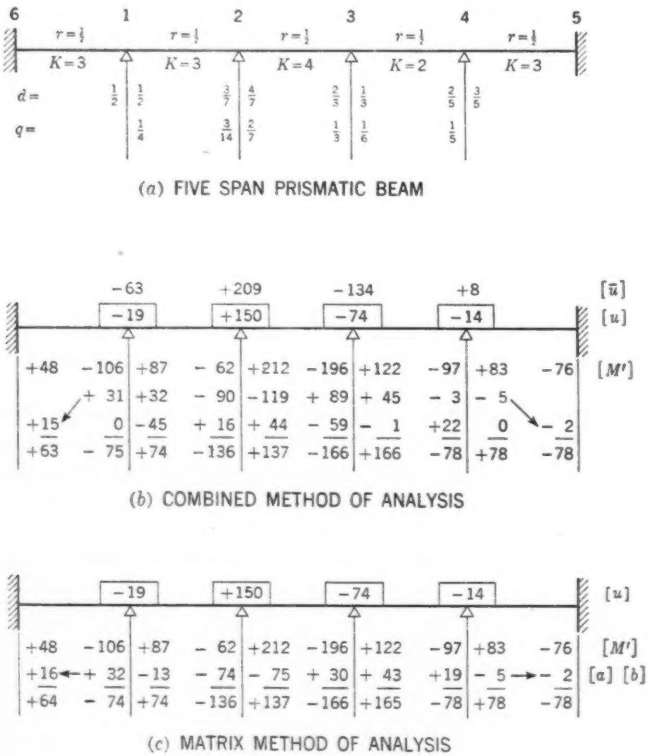
$$[Q] = -\begin{bmatrix} 0 & q_{12} & 0 \\ q_{21} & 0 & q_{23} \\ 0 & q_{32} & 0 \end{bmatrix} \quad \text{(122)}$$

In this matrix the elements have subscripts that agree with their position in the matrix. The first subscript of an element indicates its row and the second subscript indicates its column. The q-values occupy the subdiagonals and the superdiagonals whereas all other elements of the matrix are zero. The subscripts also correspond to the joint and member, the first subscript indicating the joint. It is convenient to set up $[Q]$ first and then transpose to obtain $[Q]'$.

If the coefficient matrices of Eqs. 120 are considered, the nondiagonal elements are the same as the elements of $[Q]'$. The superdiagonal enters the formula for $[C_a]$; and the subdiagonal, the formula for $[C_b]$. The diagonal elements of these coefficients are the distribution factors for the left and right ends of the members, respectively. Consequently, the only quantities that enter the evaluation of $[C_a]$ and $[C_b]$ by Eqs. 120 are primary and secondary distribution factors.

Numerical Examples

An example will be solved using each of the two variations from moment distribution which have been described. A five-span beam, having four ef-



(a) FIVE SPAN PRISMATIC BEAM

(b) COMBINED METHOD OF ANALYSIS

(c) MATRIX METHOD OF ANALYSIS

Fig. 7.—Comparison of Methods

fective joints, is shown in Fig. 7(a). Each span is prismatic, thus having the same stiffness value at either end and a carry-over factor of one half. The

stiffness values are in boxes below the center of each span. The primary distribution factors are shown for each joint and below them the secondary factors. The matrix [Q] is given by

$$[Q] = -\begin{bmatrix} 0 & q_{12} & 0 & 0 \\ q_{21} & 0 & q_{23} & 0 \\ 0 & q_{32} & 0 & q_{34} \\ 0 & 0 & q_{43} & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1/4 & 0 & 0 \\ 3/14 & 0 & 2/7 & 0 \\ 0 & 1/3 & 0 & 1/6 \\ 0 & 0 & 1/5 & 0 \end{bmatrix} \ldots (123)$$

Transposing and converting to decimals,

$$[Q]' = -\begin{bmatrix} 0 & 0.214 & 0 & 0 \\ 0.250 & 0 & 0.333 & 0 \\ 0 & 0.286 & 0 & 0.200 \\ 0 & 0 & 0.167 & 0 \end{bmatrix} \ldots\ldots\ldots\ldots (124)$$

This matrix must be used in either of the two methods of analysis.

A set of fixed-end moments are assumed as shown in Fig. 7(*b*). The unbalanced joint moments which form the components of the vector [u] are noted in the box over each joint. Using Eqs. 116 and 117, the components of the transformed vector $[\bar{u}]$ may be conveniently computed as shown in Table 1. Each column is obtained by multiplying the previous column by [Q]'.

TABLE 1.—CALCULATION OF $[\bar{u}]$

[Q]'				[u]	[u']	[u'']	[u''']	[u'''']	$[\bar{u}]$
0	−0.214	0	0	−19	−32	−6	−5	−1	−63
−0.25	0	−0.333	0	150	29	21	5	4	209
0	−0.286	0	−0.2	−74	−40	−11	−7	−2	−134
0	0	−0.167	0	−14	12	7	2	1	8

The last column is obtained by adding all the previous columns. These transformed unbalanced moments are shown in Fig. 7(*b*) just above the actual values. The application of one cycle of moment distribution to the components of $[\bar{u}]$ yields the solution as shown.

The second method of solution requires the computation of the terms of the power series in [Q]'. These terms are computed as follows:

$$[I] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (125a)$$

$$[Q]' = -\begin{bmatrix} 0 & 0.214 & 0 & 0 \\ 0.25 & 0 & 0.333 & 0 \\ 0 & 0.286 & 0 & 0.2 \\ 0 & 0 & 0.167 & 0 \end{bmatrix} \ldots\ldots\ldots\ldots (125b)$$

$$\{[Q]'\}^2 = \begin{bmatrix} 0.054 & 0 & 0.071 & 0 \\ 0 & 0.149 & 0 & 0.067 \\ 0.072 & 0 & 0.129 & 0 \\ 0 & 0.048 & 0 & 0.033 \end{bmatrix} \qquad (125c)$$

$$\{[Q]'\}^3 = -\begin{bmatrix} 0 & 0.032 & 0 & 0.014 \\ 0.038 & 0 & 0.061 & 0 \\ 0 & 0.052 & 0 & 0.026 \\ 0.012 & 0 & 0.022 & 0 \end{bmatrix} \qquad (125d)$$

$$\{[Q]'\}^4 = \begin{bmatrix} 0.008 & 0 & 0.013 & 0 \\ 0 & 0.025 & 0 & 0.012 \\ 0.013 & 0 & 0.022 & 0 \\ 0 & 0.009 & 0 & 0.004 \end{bmatrix} \qquad (125e)$$

Each matrix is obtained by multiplying the preceding matrix by $[Q]'$. Adding these matrices,

$$[I] + [Q]' + \{[Q]'\}^2 + \{[Q]'\}^3 + \{[Q]'\}^4$$
$$= \begin{bmatrix} 1.062 & -0.246 & 0.084 & -0.014 \\ -0.288 & 1.174 & -0.394 & 0.079 \\ 0.085 & -0.338 & 1.151 & -0.226 \\ -0.012 & 0.057 & -0.189 & 1.037 \end{bmatrix} \qquad (126)$$

The products $[K_a][D]^{-1}$ and $[K_b][D]^{-1}$ are given by

$$[K_a][D]^{-1} = \begin{bmatrix} 0.5 & 0.214 & 0 & 0 \\ 0 & 0.571 & 0.333 & 0 \\ 0 & 0 & 0.333 & 0.2 \\ 0 & 0 & 0 & 0.6 \end{bmatrix} \qquad (127a)$$

and

$$[K_b][D]^{-1} = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.25 & 0.429 & 0 & 0 \\ 0 & 0.286 & 0.667 & 0 \\ 0 & 0 & 0.167 & 0.4 \end{bmatrix} \qquad (127b)$$

The diagonal elements of these matrices are the left-end and the right-end primary distribution factors. The nondiagonal elements are taken directly from the matrix $[Q]'$ in the preceding computation (Eq. 125*b*). The coefficient matrices $[C_a]$ and $[C_b]$ are then computed (see Eq. 119):

$$[C_a] = -\begin{bmatrix} 0.5 & 0.214 & 0 & 0 \\ 0 & 0.571 & 0.333 & 0 \\ 0 & 0 & 0.333 & 0.2 \\ 0 & 0 & 0 & 0.6 \end{bmatrix} \begin{bmatrix} 1.062 & -0.246 & 0.084 & -0.014 \\ -0.288 & 1.174 & -0.394 & 0.079 \\ 0.085 & -0.338 & 1.151 & -0.226 \\ -0.012 & 0.057 & -0.189 & 1.037 \end{bmatrix}$$

$$= -\begin{bmatrix} 0.469 & 0.128 & -0.042 & 0.010 \\ -0.136 & 0.558 & 0.158 & -0.030 \\ 0.026 & -0.101 & 0.346 & 0.132 \\ -0.007 & 0.034 & -0.113 & 0.622 \end{bmatrix} \qquad (128a)$$

and

$$[C_b] = - \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.25 & 0.429 & 0 & 0 \\ 0 & 0.286 & 0.667 & 0 \\ 0 & 0 & 0.167 & 0.4 \end{bmatrix} \begin{bmatrix} 1.062 & -0.246 & 0.084 & -0.014 \\ -0.288 & 1.174 & -0.394 & 0.079 \\ 0.085 & -0.338 & 1.151 & -0.226 \\ -0.012 & 0.057 & -0.189 & 1.007 \end{bmatrix}$$

$$= - \begin{bmatrix} 0.531 & -0.123 & 0.042 & -0.007 \\ 0.142 & 0.442 & -0.148 & 0.030 \\ -0.026 & 0.110 & 0.655 & -0.128 \\ 0.009 & -0.034 & 0.117 & 0.377 \end{bmatrix} \ldots\ldots\ldots (128b)$$

The correction vectors can now be written as

$$[a] = \begin{bmatrix} -0.469 & -0.128 & 0.042 & -0.010 \\ 0.136 & -0.558 & -0.158 & 0.030 \\ -0.026 & 0.101 & -0.346 & -0.132 \\ 0.007 & -0.034 & 0.113 & -0.622 \end{bmatrix} [u] \ldots\ldots (129a)$$

and

$$[b] = \begin{bmatrix} -0.531 & 0.123 & -0.042 & 0.007 \\ -0.142 & -0.442 & 0.148 & -0.030 \\ 0.026 & -0.110 & -0.655 & 0.128 \\ -0.009 & 0.034 & -0.117 & -0.377 \end{bmatrix} [u] \ldots\ldots (129b)$$

The components of the correction vectors are thus expressed as linear functions of the unbalanced moments. The elements of the coefficient matrices are dependent only on the physical characteristics of the beam. In Fig. 7(*c*) the same fixed-end moments and unbalanced moments as were assumed in the previous solution are shown. The components of the correction vectors, as computed from Eqs. 129, are written directly below the fixed-end moments. Direct addition gives the final solution. If solutions for additional loading conditions are desired, it should be apparent that the solutions could be computed very quickly from Eqs. 129.

Conclusion

The linear equations that govern continuous beams have been expressed and solved in the notation of matrix algebra. A complete algebraic statement of the method of moment distribution has been given. Matrix computation methods have been illustrated. The problem treated in this paper is the most elementary type of indeterminate structure. The joints have only one degree of freedom. This limitation in the physical scope of the paper was considered desirable in order that the emphasis could be placed on the methods and concepts of matrix algebra.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CAVITATION IN HYDRAULIC STRUCTURES A SYMPOSIUM

Discussion

By Duff A. Abrams

Duff A. Abrams,[15] M. Am. Soc. C. E.[15a]—The third Symposium paper, by Mr. Warnock, consists mostly of a recital of some of the minor ailments that have been encountered by the Bureau of Reclamation, U. S. Department of the Interior (USBR), since about 1910, in the operation and maintenance of some of its hydraulic structures. The treatment of a "Boulder Dam Spillway Tunnel" could have been expanded to advantage, in proportion to its importance.

Boulder Dam Spillway Tunnel.—The considerable damage to a Boulder Dam spillway tunnel, discovered on December 12, 1941, is attributed to "cavitation" by Mr. Warnock, although it is apparent that this explanation is highly speculative. The same damage has been attributed by earlier USBR writers to "erosion" (28) and to "concrete wear" (77). The only evidence given to support this claim for "cavitation" is: "The misalinement is defined by the position of the rope in Fig. 44." In the writer's opinion, a photograph of a rope is not very satisfying as scientific evidence; especially in view of certain other considerations:

1. The "misalinement" shown may have been due to any one of a number of causes that had nothing to do with the damage to the concrete lining.
2. The "misalinement" may have occurred after the damage.
3. An official report of the USBR published in 1938 stated that there was no misalinement in the Boulder Dam spillway tunnels (78) (78*a*):

> "* * * The Boulder Dam spillway system is designed with the expectation of obtaining practically streamline flow in the lower tunnel sections where maximum velocities occur. Comprehensive specification provisions

Note.—This symposium was published in September, 1945, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: December, 1945, by C. A. Mockmore; February, 1946, by Joseph N. Bradley; March, 1946, by J. M. Robertson, and Fred W. Blaisdell; April, 1946, by John S. McNown; May, 1946, by James W. Ball, and Fred Locher; and June, 1946, by Carl E. Kindsvater, and E. R. Van Driest.

[15] Cons. Engr., New York, N. Y

[15a] Received May 1, 1946.

for accuracy of alinement and rigid control of concrete manufacture and placement help to insure this." * * *

"Since considerable care was taken in designing the shapes of the spillway transitions to eliminate negative pressures or vacuums, it is not expected that erosion will occur due to cavitation."

4. The damage was in exact accord with the teachings of model tests of concrete erosion, published by the USBR prior to damage.

5. Apparently previous inspections during seven years after completion had revealed no misalinement in the tunnel lining.

6. Even if one wished to accept the hypothesis of "cavitation," a statement by Mr. Warnock makes it impossible to do so:

"Actually, the coat of black waterproofing and mineral deposit was intact in many places, showing no effect of direct scouring by the high-velocity water immediately above and below the eroded area."

Mr. Warnock continuously keeps before the reader the merits of many different types of model tests. The word "model" ("laboratory studies," "laboratory investigations," etc.) is used twenty times; but in all his comment on model testing he omits one of the most significant "model-prototype" relations to be found in the literature. By whatever name it is designated, the performance of the Boulder Dam spillway was exactly in accord with the teachings of model tests made by the USBR ten years earlier.

Prior to the construction of the dam, two types of model tests were conducted: (*a*) Erosion of concrete blocks by a jet of water at high velocity; and (*b*) hydraulic model tests of various spillway features.

Before discussing the model tests, it may be well to review what happened. Boulder Dam is a concrete arch-gravity structure, 727 ft high, completed by USBR in 1935. Two spillways were built. Water discharges into a side-channel spillway and then into a concrete-lined tunnel (50-ft inside diameter), inclined to the horizontal at 50°. Water storage in Lake Mead (above Boulder Dam) began in 1935; but it was six years before an opportunity arose to compare the performance of the "prototype" with the models. During four months in 1941, about 13,500 cu ft per sec was discharged by the Arizona spillway. This spillway was designed for a capacity of 200,000 cu ft per sec. The quantity discharged was about 7% of the rated capacity, and was considered so insignificant that no hydraulic model tests were made with less than equivalent discharges of about 40,000 cu ft per sec. Nevertheless, the discharge for a short time of 7% of rated capacity produced the following rather startling results:

(*a*) Ripped a hole 115 ft long and 33 ft wide at the bottom of the drop in the 50-ft circular tunnel;

(*b*) Washed away a seven-year-old concrete lining to depths of as much as 25 ft;

(*c*) Excavated a hole 25 ft deep in the native rock; and

(*d*) Repairs required seventeen months, and cost $250,000.

The principal feature of the repairs was the placing of 1,460 cu yd of concrete.

It may seem strange that the concrete lining was as much as 25 ft thick; but the bottom of the drop was in a "tunnel plug" built to seal off the upstream part of the original 50-ft diversion tunnel.

Model Tests of Concrete Erosion.—In 1931, Arthur Reuttgers and B. W. Steele, Assoc. M. Am. Soc. C. E., wrote "Tests of Concrete Blocks Subjected to High Water Jet Velocities." These tests were made two years before the first concrete was placed in Boulder Dam. The complete report was reprinted in 1938 (78*b*). The original purpose of the tests was stated as follows (78*c*):

> "Experiments were made to determine the resistance of concrete to erosive action of clear water flowing at extremely high velocities. Although the experiments were not part of the laboratory tests of hydraulic properties of spillways, they were definitely related to the practical performance of the spillway structures. Since no previously existent structures had been known to carry water at velocities of nearly 180 feet per second the investigations were of considerable importance."

Attention is directed particularly to the statements made seven years after the tests and three years after the completion of Boulder Dam to the effect that:

(*a*) The tests "were made to determine the resistance of concrete to erosive action of clear water,"

(*b*) They "were definitely related to the practical performance of the spillway structures,"

(*c*) "* * * the investigations were of considerable importance."

In 1938 the USBR announced the titles of forty-one volumes of reports that were in process of publication, giving technical details of all features of the design and construction of Boulder Dam and appurtenant works. Seven of these books are devoted to the $500,000 research program on cement and concrete. Perhaps it was some measure of the relative importance of the subject that the report on "Model Studies of Spillways" (78) was the first to appear of those having to do with concrete.

Sixteen small blocks of concrete, 18 in. square and 6 in. thick, were made in Denver, Colo. The mix was 1:2:4 using sand and gravel graded to $1\frac{1}{2}$ in.; water-cement ratio, 0.63, by weight; and slump from 3 in. to 5 in. The blocks were cast on smooth, plane steel plates. The type of cement and the strength of concrete were not reported.

At ages of from 40 days to 138 days the blocks were subjected to a stream of water at velocities of from 100 ft per sec to 175 ft per sec. The stream was applied in a downward direction by means of a fixed nozzle that tapered from 4-in. inside diameter to 1 in. in 20 in. The test block was bolted at the required angle, so that the stream impinged on the surface 15 in. from the nozzle. Generally the stream was applied at an angle of 90° to the bottom surface of the blocks, although angles of 5°, 30°, and 45° were also used. On some of the blocks, other methods were used, such as playing the stream in a circular slot cast in the concrete, or against an offset cast in the concrete. The treatment was continued, from a few hours to two days—in one instance for five weeks.

Typical Results of Model Tests of Concrete Erosion.—Block A–2 was subjected to water at a nozzle velocity of 175 ft per sec, at an angle of 90° for five days. It has been stated (78*d*) that this block was:

"Unintentionally tested on upper, trowelled surface (as cast). The mortar surface was worn off in the shape of an annular ring."

The sections of Boulder Dam spillways that are subjected to the highest velocities have unformed surfaces; hence the foregoing test, made contrary to plan, gave the only information in the entire report that is directly applicable to the conditions at Boulder Dam. The remaining tests on block A–2 were made on the "formed" surface, but gave more tangible information on the dimensions of the "annular ring." At the end of a three-week test:

"Erosion was confined to strip 1-inch wide on outside of a circle of approximately 4-inch diameter. Pits less than ½-inch deep" (78*d*).

After block A–2 had been under test at 175 ft per sec at 90° for five weeks: "Largest hole in block 1-inch deep and 12-inches square at the bottom. Rest of erosion less than ½-inch deep" (78*d*). It is not clear how a nozzle that discharged a scattered spray 4 in. in diameter could form a hole in the concrete "12-inches square at the bottom." This question would not ordinarily be important, but in this instance the information appears in an official USBR report and concerns data on which Boulder Dam was designed and built.

The tests on block A-2 were the most extended in the program. They demonstrated fully the fundamental error of applying a scattered spray, rather than a concentrated jet of water. At the same time, they showed the inability of this concrete to withstand even this mild type of erosion. In testing block F–1, the stream from the nozzle entered tangentially a "12-inch semicircular groove" cast in the concrete. After one day at 175 ft per sec: "Maximum depth of cavitation approximately ⅛ inch."

Because of entrainment of air, scattering of water, rebound, etc., the tests made with this setup were entirely inadequate when applied to Boulder Dam spillways. When the stream was applied at small angles the treatment was still less significant. Notes from the report on three blocks tested with the nozzle set at 5° to the test surface are presented in Table 2. Of course, "very little erosion" resulted from this scattered spray, which did not remotely resemble operating conditions at Boulder Dam. These notes are typical of the observed data from the model tests of concrete erosion; but the destructive effects of these innocuous treatments were overlooked entirely in the design and construction of Boulder Dam.

TABLE 2.—Tests on Concrete Blocks Subjected to High Water Jet Velocities

Block	Velocity[a]	Days	Observed data
A-1..	125	1	No appreciable erosion
A-1..	150	1	No appreciable erosion
A-1..	175	9.6	Small rough spot, some enlargement of surface pits
B-1..	175	2	Very little erosion
C-1..	175	2	Very little erosion

[a] Feet per second.

Comparison of Model and Prototype.—Although it is apparent that the model tests of concrete erosion were inadequate to represent conditions at

Boulder Dam, it is extremely interesting to note the almost perfect agreement between phenomena for model and "prototype":

Concrete model tests, 1931	Prototype tests, 1941
Under jet velocity of 175 ft per sec; cavitation of $\frac{1}{8}$ in. in one day; wore an annular ring in five days; a hole "1-inch deep and 12-inches square" in five weeks.	At a velocity estimated by Mr. Warnock of "at least 150 ft per sec," and at 7% of design capacity, 3,000 tons of rock and concrete was washed away in four months.

The superior quality of the spillway concrete, as a result of its age of seven years, was more than counterbalanced by the longer period of test. Ten years before the "prototype" was used, the model tests showed that concrete could not withstand even this mild treatment. The model tests predicted exactly the nature but not the full extent of the damage that was to occur in the spillway tunnel.

Concrete of Controlled Quality.—The two quotations (78) (78*a*) given near the beginning of this discussion, published three years after the completion of Boulder Dam, assured the profession that: "Comprehensive specification provisions for accuracy of alinement and rigid control of concrete manufacture and placement help to insure this [streamline flow at critical sections]"; and that: "* * * it is not expected that erosion will occur due to cavitation." Mr. Warnock now claims that the spillway failure was due to the very causes that were renounced years earlier:

> "Imperfections in the concrete, such as rock pockets, cold joints, porous areas, lack of bond, etc., all made the concrete more vulnerable to this attack by impingement. Furthermore, the impingement of the high-velocity water on any exposed joints would cause the energy in the water to be converted from velocity head to pressure head. This pressure was probably transmitted through the planes of weakness in the construction joints caused by lack of proper horizontal joint cleanup prior to placement of new concrete."

Horizontal Construction Joints.—The damage described by Mr. Warnock is attributed to the horizontal construction joints. Nearly three years prior to the discovery of the damage, the writer called attention to the defects of the water-air-gun method of joint cleanup that was used at Boulder Dam, and pointed out that the method (79): (*a*) Was dangerous, (*b*) should never have been specified, and (*c*) should be strictly prohibited. In response to this criticism, R. F. Blanks, M. Am. Soc. C. E. (80), made a vigorous defense of this method, to which Mr. Warnock now attributes the severe damage of the spillway tunnel.

That reservoir water is finding its way freely into or through the horizontal construction joints at Boulder Dam is amply attested by the following:

(1) In 1940 Tom C. Mead (81) reported that:

> "Knowledge of the composition of the seepage water should help also in understanding the reason for the deposition of calcium carbonate in the foundation drains."

The mechanism here is quite plain. Cold reservoir water, about 60° F, enters the drains where they intercept the horizontal joints. In passing through the joints, the cold water became saturated with calcium hydroxide from the cement. Water in the drains is heated to about 100° F by the warm rock. Warm water cannot hold in solution as much calcium hydroxide as cold water; hence the excess hydroxide is precipitated in the drain holes, and gradually becomes carbonated from combination with the carbon dioxide in the water. It seems extremely likely that these drains will soon become entirely inoperative as a result of this deposition of carbonate.

(2) In 1945 Clarence Rawhouser, Assoc. M. Am. Soc. C. E. (82), called attention to "drilling of additional drain holes" under the middle of Boulder Dam. This was made necessary, no doubt, by the development of excessive hydrostatic uplift, due to water from the lake passing freely into the horizontal joints.

(3) In the same discussion Mr. Rawhouser gave data on one thermometer embedded in the concrete near the middle of the bottom of the dam which showed a most rapid rise in temperature for the first two years after artificial cooling was discontinued; then there was a much smaller rise for three years, followed by a drop of about 10° F during the last five years. The "unexplained" behavior of this thermometer was probably due to the gradual cooling of the hot concrete by the cold reservoir water passing freely through the joints.

(4) Mr. Warnock concludes that: "The concrete was probably dislodged in quite large pieces." If so, this was facilitated, no doubt, by hydrostatic uplift from lake water in the joints.

(5) In two places in his paper Mr. Warnock mentions the "mineral deposits" encountered in the area of the failure of the tunnel lining. These deposits were undoubtedly due to lime from the cement carried to the surface by lake water passing through the horizontal construction joints.

The Durability of Boulder Dam Concrete.—One of the most disturbing features of the damage in the Boulder Dam spillway tunnel was a report issued by representatives of the USBR. At the same time that the USBR was spending $250,000 to repair the damaged tunnel, it was informing the profession that (77):

> "Bureau of Reclamation tests indicated that concrete of ordinary proportions and controlled quality will withstand any erosive action likely to be produced in Boulder Dam spillways."

Both the concrete tests and the spillway had previously demonstrated that "concrete of ordinary proportions and controlled quality" would not do what is here claimed. The foregoing statement was published more than a year after it was known that 3,000 tons of concrete and rock had been washed out of one of the Boulder Dam spillways the first time it had been required to discharge 7% of its design capacity.

Summary.—The concrete model testing done for the design of the Boulder Dam spillways affords one of the most remarkable opportunities for "model-prototype" comparisons in engineering literature. The results of the concrete block tests in 1931 were completely corroborated ten years later by the first trial run of a Boulder Dam spillway. The model showed decided pitting at one

day, and serious erosion after five weeks of exposure to a scattered spray of water at 175 ft per sec. In the spillway, 3,000 tons of concrete and rock were washed away, in four months, by water equivalent to 7% of the rated capacity.

The damage to the spillway tunnel was primarily due to erosion of the concrete (as had been shown by the model tests) and secondarily to hydrostatic uplift from reservoir water that found its way through the horizontal construction joints. This effect under a discharge of 7% of rated capacity, has a significance much more important than merely an interesting example of concrete erosion. It demonstrates that: (*a*) Concrete cannot withstand the erosive action of water flowing in these spillways; and (*b*) this spillway, having failed to withstand a discharge of 13,500 cu ft per sec, is entirely inadequate to discharge the 200,000 cu ft per sec for which it was designed.

In spite of the plain failure of the concrete in the model tests of erosion, and more than a year after the experience at the spillway tunnel representatives of the USBR issued a misleading statement on the capability of this concrete to withstand the service conditions to which it is exposed. This statement is almost certain to lure other engineers into the same types of costly and dangerous decisions that have occurred at Boulder Dam.

Bibliography.—

(28) "Erosion Causes Invert Break in Boulder Dam Spillway Tunnel," by Kenneth B. Keener, *Engineering News-Record*, November 18, 1943, p. 762.

(77) "Report on Significance of Tests of Concrete and Concrete Aggregates—Wear Resistance of Concrete," by L. H. Tuthill and R. F. Blanks, A.S.T.M., 1943, p. 38.

(78) "Model Studies of Spillways," *Bulletin VI–1*, Boulder Canyon Project Final Reports, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo., 1938, p. 173. (*a*) p. 185. (*b*) p. 163. (*c*) p. 9. (*d*) p. 180.

(79) *Journal*, A.C.I., June, 1939, p. 188–1.

(80) *Ibid.*, p. 188–7.

(81) "Technical Investigations at Boulder Dam," by Tom C. Mead, *Reclamation Era*, June, 1940.

(82) *Journal* (Supplement), A.C.I., November, 1945, p. 348–23.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE PLANNING OF AERIAL PHOTOGRAPHIC PROJECTS

Discussion

BY MARSHALL S. WRIGHT, AND ROBERT H. RANDALL

MARSHALL S. WRIGHT,[6] ESQ.[6a]—Since 1936 the U. S. Department of Agriculture has followed a procedure, whereby all proposed aerial photographic projects are first submitted to the Office of the Secretary for approval before photographic operations are initiated either directly or by contract. During the period from 1938 to December, 1941 (Pearl Harbor), all applications to initiate work had to be accompanied by an "Engineer's Estimate" of cost, in the manner the author has indicated. During World War II the requirement that an "Engineer's Estimate" be submitted to the Secretary's Office along with all applications to initiate aerial photographic operations was discontinued, due mostly to the fact that aerial photographic operations were greatly curtailed at the request of the War and Navy departments. Furthermore, it was practically impossible to prepare cost estimates, because of delays and unforeseen restrictions on normal operations.

A resumption of aerial photographic activities of the magnitude in effect during the period from 1936 to 1941, whereby practically two thirds of the area of the United States was photographed to rigid federal specifications by commercial contractors, would necessitate the continuance of a thorough check of estimated costs as balanced against contract prices to assure the government a commensurate return on its investment; and, what is equally important, would require a constant study to assure that the most efficient methods are employed. This would also necessitate, on the part of the contracting officer, a thorough knowledge of all improvements in photographic and flying equipment, field and laboratory technique, and an intimate knowledge of the facilities of all contractors.

In the inauguration of the cost-analysis method outlined by the author, possibly the two greatest concrete benefits that were derived were (1) the

NOTE.—This paper by F. J. Sette was published in March, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September 1946, by T. W. Norcross, and James M. Cultice.

[6] Technical Asst. to the Chf., Office of Plant and Operations, U. S. Dept. of Agriculture, Washington, D. C.

[6a] Received July 16, 1946.

preparation and subsequent placing in the hands of every potential commercial contractor the indicated map (Fig. 2) of the United States showing, by regions, areas of comparable weather condition expectancies; and (2) as a consequence of benefit (1) the over-all planning of seasonal work to permit a contractor to be awarded areas in the northern part of the United States during the summer months. The map (benefit (1)), the cost of which probably could not be justified by any one single contractor, was a major contribution to the entire aerial photographic industry. Admittedly, it did "do something about the weather"; and, most directly, it was probably the greatest contributing factor toward an indicated reduction of approximately $1.00 per sq mile (compare 1938 with 1939 in Table 10).

Because of benefit (2), the contractor was enabled to discontinue this work in the fall, and move to the southern states and continue photographic operations in other contracted areas. The result was evident in lower bid prices because the contractor could utilize his personnel and equipment on a yearlong basis.

One phase that the author has not mentioned is that the Congress of the United States included in the Agricultural Appropriation Act for 1938 the following provisions which permit the sale, to the public, of photographic reproductions:

> "The Secretary may furnish reproductions of such aerial or other photographs, mosaics, and maps as have been obtained in connection with the authorized work of the Department to farmers and governmental agencies at the estimated cost of furnishing such reproductions, and to persons other than farmers at such prices (not less than estimated cost of furnishing such reproductions) as the Secretary may determine, the money received from such sales to be deposited in the Treasury to the credit of the appropriations charged with the cost of making such reproductions. This section shall not affect the power of the Secretary to make other disposition of such or similar materials under any other provisions of existing law."

As evidence of the great usefulness of aerial photography in the solution of all land-use problems, the Department has sold approximately $850,000 worth of aerial photographic reproductions to states, public utility corporations, and individuals, excluding the great use for them by other federal agencies.

ROBERT H. RANDALL,[7] M. ASCE.[7a]—Both the user and the producer of aerial photography will profit by this paper. The studies described were undertaken at a time when the aerial photographic program of the United States Department of Agriculture (USDA) was in its early stages. The procedures that developed from them unquestionably saved considerable money in the first year's operation. Commercial organizations working on contracts with the USDA also undoubtedly profited by these studies. Originally undertaken for the use of the USDA in administering its own photographic program, the results of this pioneer work continue to be of use not only to the USDA but to all users of aerial photography in the federal government and to all concerns supplying aerial photographs.

[7] Chf. Examiner, Surveying and Mapping, U. S. Bureau of the Budget, Washington, D. C.

[7a] Received July 26, 1946.

The studies reported in this paper constituted a program of research which had never been attempted on such a scale, by aerial photographers, individually or collectively. Reduced to usable and convenient form, they furnished statistics on weather conditions by which, for the first time, the USDA could plan its photographic requirements in different areas in such a way as to take advantage of the best weather conditions. This development was in itself an economy to the government. Furthermore, the photographic contractors were enabled to forecast conditions under which they would be required to operate, thus lowering their bid prices. During the first year that this analysis of weather conditions was available to the contractors, it is estimated that the USDA saved about $500,000.

In addition to the direct economies to the government and to the contractors by the availability of information on weather conditions, the studies described in this paper produced a second material, although less direct, benefit: The USDA was able to inaugurate a practice of making detailed estimates for each photographic job requested by any of its bureaus or agencies. This procedure of preparing estimates, and of checking the bids received against them, constituted an improvement in administrative practice, which, although no longer followed by the USDA in the degree of refinement that was observed in the earlier days of the photographic program, is still followed in general not only by the USDA but also by other federal departments and establishments.

It is impossible, of course, to determine the extent to which this administrative practice of USDA influenced other federal agencies outside the department. It is a fact, nevertheless, that all agencies procuring aerial photographs for the purpose of making maps or charts now recognize the common-sense value of careful advance estimating. In 1941, when these agencies (under the suggestion of the Bureau of the Budget in the Executive Office of the President) developed a uniform system for accounting and reporting their performance and costs, they included advance estimating for aerial photographic projects. In this system the planning of aerial photography can be separated into five steps: (1) Determination of limits of project; (2) consultation of records, to ascertain what maps and photographs are already available; (3) determination of scale or flying height; (4) preparation of estimates of time and cost; and (5) administrative and technical clearance to conduct the work—that is, to determine the availability of funds, obtain final authorization for expenditures, etc.

Although the aerial photographic requirements of the USDA and other federal agencies were many, at the time that Mr. Sette's studies were made, the aerial photographic interests of the United States are now even greater. During World War II the federal government photographed some 15,000,000 sq miles in various parts of the world; and, with this and other available cartographic material, compiled and published aeronautical charts covering all the land areas of the planet. Besides aeronautical charts, the armed forces also produced topographic maps on various scales in many parts of the world. To revise and improve these as the national interest may indicate, and to keep them up to date, implies a continued national interest in aerial photography in respect to not only the continental United States but the remainder of the world as well. The sensible procedures described in this paper should continue to contribute to the efficiency and economy with which this work may be done.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SOME THOUGHTS ON ENGINEERING EDUCATION

Discussion

By Robert O. Thomas, Clement C. Williams, N. W. Dougherty, H. A. Wagner, M. E. McIver, and Scott B. Lilly

Robert O. Thomas,[14] Assoc. M. ASCE.[14a]—The parts of this paper that deal with the practice of engineering and the future market for engineering services are of especial interest. The subject should be given the widest possible consideration among all members of the profession. It should be kept continually alive by extended studies and reports in the technical press so that the profession as a whole may be made aware of the dangers confronting it and so that they may be informed of the measures which must be formulated and practiced in order to arrest the present apparent trend toward artisan rank.

For a number of years, the writer has been aware of trends in the practice of engineering which have been so well illustrated by Mr. Baker, and to which attention was called by the writer[15] in 1940.

A study of past annual reports of the Society has developed information of considerable interest relating to the growth, both of the Society itself and of the engineering profession in general. The disparity between the number of new members admitted each year since 1929 and the total gain in Society membership is very striking.

Table 7 illustrates vividly the fact that it is necessary for the Society to admit three new members to secure a permanent gain of only one member. Such a condition in a profession is one which invites extensive research into the causes underlying the movement of men into and out of the profession. Engineering is not a trade, nor is it a type of clerical work that can be taught quickly to an untrained candidate for a job. The 11,126 lost members represent at least four years of study per member—a huge investment in education

Note.—This paper by Donald M. Baker was published in April, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1946, by E. S. Boalich, Russell C. Brinker, I. Oesterblom, Samuel T. Carpenter, Lynn Perry, and L. E. Grinter.

[14] Engr., Edward R. Bowen, Cons. Engr., Los Angeles, Calif.

[14a] Received July 15, 1946.

[15] "Engineering Trends in California," by R. O. Thomas, *Civil Engineering*, July, 1940, p. 466.

(individual) and educational facilities (institutional). They represent a group which certainly, at one time, expected to follow the profession of engineering as a life career and prepared accordingly.

Over the sixteen-year period, the actual loss, not counting deaths, amounted to a total of 8,485 members. Each member should be regarded as an avoidable loss—either avoidable through the retention of the men in the profession of engineering and in the Society, or avoidable through more enlightened guidance in the precollege period to those seeking to follow the profession. It must also be remembered that this group of lost engineers represents only those who are able to number themselves among the group of former members of the Society; and it does not include the loss of the considerable number of engineers with which the Society has never come in contact. In considering the loss to the Society, several questions present themselves for discussion. What has turned these 8,485 members away from the Society? Have they left the practice of engineering or have they merely left the Society? If they are still following an engineering career, why did they leave the Society? How could the profession as a whole (or the Society as the representative of the profession) have saved these engineers, either for engineering or for the Society? Were they unsuited to practice the profession? Was some other line of endeavor more remunerative or more attractive? Were they given opportunity to advance in the profession they had chosen, or was their progress made difficult by the lack of cooperation, of fellowship, or of understanding and assistance given them by those who had already attained reputation and recognition in the profession? Whatever the causes, it is the writer's belief that they must be sought out and subjected to searching analysis, because it is only on the basis of truth that the profession can hope to progress.

TABLE 7.—MEMBERSHIP STATISTICS, AMERICAN SOCIETY OF CIVIL ENGINEERS, FOR THE YEAR PRECEDING A GIVEN DATE

Date (Jan. 1)	Total members	New members	Losses	Net gain	Deaths
1929.....	13,315	1,066	558	508	135
1930.....	13,823	1,139	565	574	138
1931.....	14,397	1,055	524	531	140
1932.....	14,928	753	696	57	145
1933.....	14,985	534	580	−46	170
1934.....	14,939	757	786	−29	170
1935.....	14,910	923	764	159	165
1936.....	15,069	1,142	1,110	32	170
1937.....	15,101	1,123	765	358	150
1938.....	15,459	1,147	812	335	160
1939.....	15,794	1,240	1,034	206	177
1940.....	16,000	1,422	760	662	157
1941.....	16,662	1,399	623	776	179
1942.....	17,438	1,471	555	916	190
1943.....	18,354	1,576	500	1,076	190
1944.....	19,430	1,497	494	1,003	205
Total..	....	18,244	11,126	6,115	2,641

To secure an admittedly approximate answer to the question of the composition in membership of those leaving the Society, the writer made a study of the membership as given in the ASCE *Yearbook* for 1945. A random 10% sampling was made which resulted in determining that 35.3% of the members listed (a total of 7,250) had been members of the Society in 1929. The 1929 membership was 13,315, from which it may be seen that 6,065 of the members of the Society in that year were no longer connected with the Society. Inasmuch as there were only 2,043 Juniors in 1929, of which a considerable number must have advanced to corporate grade; and, since there have been only 2,641 deaths to date, including both those who were members in 1929 and those

joining since, it is readily apparent that a major proportion of those lost to the Society must of necessity have been corporate members. Why have these men, who had followed the profession of engineering for an appreciable period and who had advanced to responsible positions in the profession, left the Society? The answer to this question is vital. Even if these men have left the profession, both professional and sentimental instincts should have kept their membership alive and retained their interests. If they have not left the profession, why are they no longer members of the Society?

To establish some verification for the foregoing data from an independent source, the writer had recourse to the admirable chart prepared by Arthur Richards,[16] M. ASCE, from which Table 8 was prepared. The chart included the results of case studies of 5,000 civil engineers by the Society for the Promotion of Engineering Education; 9,000 mechanical engineers by the American Society of Mechanical Engineers; and 15,000 highway engineers—a total of 29,000 case histories. The writer has applied the diagram to the theoretical histories of groups of 100 engineers beginning practice in successive five-year periods. The data show that of 633 engineers of various ages, active in their profession, there will be only 272 after a lapse of fifteen years, the remaining 361 having either left the profession or died. This represents a loss, over a fifteen-year period, of 57% of the original total.

TABLE 8.—MORTALITY STATISTICS IN THE ENGINEERING PROFESSION

Age	Number in the profession	Number remaining after fifteen years	Loss
25	100	97	3
30	135	66	69
35	131	44	87
40	97	30	67
45	66	20	46
50	44	10	34
55	30	5	25
60	20	..	20
65	10	..	10
Total	633	272	361

The number of engineers under the age of 35 is given in Table 8 as 366, or 58% of the total practicing engineers. The large percentage of younger men, contrasting with the figure of 14.9% for the Juniors in the American Society of Civil Engineers in 1929, seems to be explained by the preponderance of highway engineers in the basic data, highway engineering being particularly attractive to younger men because of the opportunities for outdoor work. Table 9 shows a comparison of data as developed for the Society for the sixteen-year period, 1929 to 1944, and as computed from the Richards curves for a fifteen-year period.

TABLE 9.—COMPARISON OF ASCE AND RICHARDS MORTALITY DATA

Item	ASCE	Richards
Number:		
Engaged in practice of engineering	13,315	633
Remaining after fifteen years	7,250	272
Leaving during fifteen years	6,065	361
Percentage:		
Under the age of 35 (ASCE = 32 yr)	14.9	58.0
Of the original number that has been lost	45.6	57.0

Again taking into consideration the fact that a large percentage of the case histories studied by Mr. Richards applied to highway engineers (predominantly

[16] "Vocational Guidance in Engineering Lines," Am. Assn. of Engrs., 1933, p. 160.

younger men), and the assumption that engineers, taken as a whole, who are the type desiring affiliation with a professional society, are somewhat more likely to remain in their profession than are those who do not so affiliate themselves—there still remains an excellent degree of correlation between the two studies. It can probably be stated as a general principle, that between 45% and 50% of the men who train for and enter the profession of engineering leave the profession for another line of endeavor prior to the termination of their working lives, and as a further corollary (assuming that practically all who leave after the age of 45 are lost by death) it can be stated that they leave the profession in the first fifteen years.

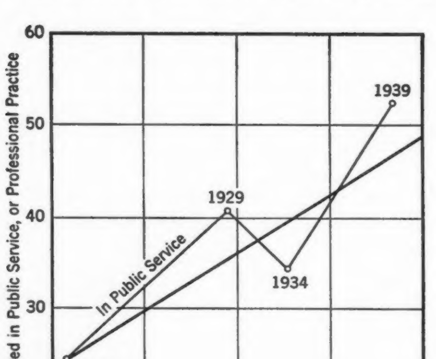


FIG. 2.—GROWTH OF PERCENTAGE OF JUNIORS, ASCE, IN CALIFORNIA, 1924–1939, AND CONCURRENT CHANGE IN EMPLOYMENT CLASSIFICATION

This brings to the fore the subject of the loss of Juniors who find themselves unable to complete the requirements for transfer to the grade of Associate Member. This question has received deep and serious study by the Society's Board of Direction, and culminated in raising the age limit for Juniors to 35 in 1942. As time goes on, apparently, it is increasingly more difficult for Juniors to secure the necessary year in responsible charge of work, and this condition is reflected in the large number of Juniors dropped yearly because they have reached the age limit. This is explained by the large number of Juniors who enter the public service field, where the opportunities for promotion, and consequently for independent responsibility, are slow in coming. Fig. 2 shows graphically the trend of Juniors of the Society toward seeking employment with the large civil service engineering departments, and perhaps accounts in some measure for their failure to qualify for advancement in the Society. It may account, also, for a considerable number of the engineers lost to the profession, as the individual becomes disappointed in his chosen work when he finds the field to advancement and responsibility almost hopelessly blocked; and he turns to other and more attractive opportunities in other fields.

From the data presented herein the following questions are evident: Are young engineers being educated, or trained, properly? If so why are they (and also an appreciable portion of older engineers) leaving the profession, or the Society? If they are not being trained properly, what should be done to reorient engineering education? In the light of the demonstrated magnitude of the numbers of engineers who leave the profession, should the course be length-

ened to include material from other major fields such as business administration, public administration, etc., to better prepare the candidate for an engineering degree not only to succeed in the practice of engineering but also in the pursuit of other occupations? Are opportunities in engineering, and the rewards concomitant with them, appropriate to the knowledge, ability, and responsibility involved? If not, what can be done to improve them? If they are, why is the nation losing the services of so many who were once engineers? The future of the civil engineering profession is acutely dependent on the solution to these and other interrelated questions.

Clement C. Williams,[17] M. ASCE.[17a]—Gratitude is due the author for his able analysis of the character of the profession and for his collection of pertinent information concerning its composition. The paper discusses engineering education from the viewpoint of the practicing engineer, namely, a commodity purchasable on specifications. It is a viewpoint that widely prevailed prior to World War I. Many professors of civil engineering, with experience limited to T-square operations, pointed their students to the drafting rooms of the American Bridge Company; professors of electrical and mechanical engineering aimed their instruction chiefly toward the apprenticeships of the General Electric Company or the Westinghouse Company. The standard practices of those companies dominated the instruction of the senior year. Engineering then was confined to applied science and the accumulated art. With the expansion of research and of graduate preparation of faculty, the situation has changed; engineering has germinated its own investigative science.

Few educators of today (1946) consider the training of young men for the requirements of present practice as the gage of their objectives. The future careers are too fortuitous in detail to warrant planning with specificity. Engineering education is more a function of the native capacities of youth and of scientific discovery than of office requirements; instructional subject matter is derived more from researches than from practice. In this new era of education, engineering has made conspicuous advances.

Although all education has a social purpose, its social adaptation is not a primary concern of the college but is a matter for later determination. The author's analogy to steel billets and subsequent products is correct but somewhat remote. A closer, but still faulty, parallel would be that college attends to the growth of a tree so that it becomes strong and without defects, but leaves the question of what timbers may be cut from the log for later circumstances to decide. Engineering education consists in cultivating the "growth" of the faculties to observe, to think, and to express thoughts, and of the personal attributes essential to good citizenship and a full life. In mental discipline, it involves an acquaintance with the basic sciences and knowledge as thought tools and material, followed by exercises in thinking concerning supposititious engineering situations—all critically compared with the more mature and tested ideas of the teacher and the textbook. Exercises in thinking are as essential to mental development as physical exercises are to bodily improvement. The

[17] Consultant in Eng. and Industrial Education, Madison, Wis.

[17a] Received July 17, 1946.

former are the crux of the learning process. The author seems to mistake these supposititious engineering exercises for an attempt to teach engineering practice. Of course, their value is enhanced by the closeness of their correspondence to actualities; and, if well chosen, they do impart a modicum of practical information. They also serve as a pedagogical device to stimulate interest, so indispensable to learning. They are of value in developing powers of ratiocination even if they relate to situations which never arise, or to situations entirely forgotten by the student, since these powers, if once ingrained, can be recalled from the subconscious mind although he may think them gone forever.

The author also seems to overestimate the maturity of student experience. What a student can absorb and assimilate depends on his previous experience. The concepts of a mature engineer cannot be grafted on to the tender experience of a youth who has had no observation of engineering work. A lecture beyond the student's clear grasp is futile, for "nothing learned, nothing taught" applies regardless of the profundity of the message.

Professions are not rigid arrangements of human services but are clusters of similar operations grouped more or less statistically according to the frequency of their occurrence. Mechanical skills fit into definite requirements, but engineering is expansible and adaptable to its social function. The sixty members of the writer's own class in civil engineering, as revealed by the annual class letter, have served society well; but relatively few of them at present are doing work closely related to the class instruction of the undergraduate years. Following the progress of hundreds of students who have passed under the writer's supervision brings the conviction that careers are affected by circumstances mainly coincidental, and that it is wholly conjectural whether any particular modification of their curricula would have improved their fortunes. Engineering education aims at a hypothetical service as a bull's-eye, but its results follow a scatter or probability pattern centered on the maximum of incidence.

It is not derogatory to engineering education, therefore, to find that those so prepared are distributed over a multitude of occupational needs in organized society. Indeed, since each particular activity cannot feasibly be accorded specialized training, this flexibility of engineering education is a social virtue. Education for the law presents a similar situation in that it prepares for a large number of ancillary vocations which contribute to successful society. In a city of 15,000, where the writer once lived, there were seventy-two lawyers—that is, persons who had been admitted to the bar—although not more than ten or a dozen practiced at court. The others were realtors, securities agents and abstracters; or were engaged in local politics, in business, or otherwise serving the community. The law touches human affairs at many points, just as applications of physical science do; hence, one may expect preparation for law, or for engineering, to be widely useful in organized society outside the strict confines of the respective professions. In medicine, by contrast, the preparation is more specific and is seldom applicable beyond health and hygiene; nevertheless, quite a few doctors become managers of business and civic leaders, as do many men who have had no higher education. From these circumstances, the inference

may be drawn that specific training for unusual eventualities does not need to be included in the professional curriculum.

The author proposes that curricula provide a broader scope of basic instruction covering the entire field of engineering and leave advanced theory for graduate study. There were many protagonists of this idea a quarter of a century ago. A number of institutions, especially smaller colleges, adopted the plan of having three years for all departments in a common curriculum with departmental differentiation in the senior year. The University of Iowa, Iowa City, adopted this scheme. The writer became dean of engineering at that university in 1926 and witnessed the change back to the usual distribution of subject matter. Some of the reasons for the reversal were (1) the graduates did not make a favorable showing when they entered apprenticeship courses; (2) the classes could not use standard textbooks; and (3) most of the faculty were dissatisfied with the unstimulating elementary instruction involved. Some of the small colleges still continue the plan. Their alumni cannot enter graduate schools directly; nor are they so acceptable to industry; and their curricula are generally not accredited by the Engineers' Council for Professional Development (ECPD). Many engineering colleges have a "general engineering" curriculum which includes about equal parts of all branches, but it has not proved superior in vocational adjustment.

A longer period than four years in the engineering college may profitably be spent by many students, either (1) in pursuing the humanities or the social studies, (2) in the pure sciences and mathematics, or (3) in the technical subjects of engineering curricula other than the student's major. Many students so continued their education during the depression years when jobs were not available, and, generally, they found the additional study profitable. However, the length of an engineering education, like other engineering undertakings, has economic limitations, which, for the present scale of compensation, seems to be four years. The students of superior ability find postgraduate study, even through the doctorate, advantageous. For students of the lower half of the class, graduate study, so far as the writer has observed, seldom advances the engineer's professional success.

As a matter of fact, the preparation of engineering students is more general than most industrial personnel directors and practicing engineers are aware. Students have some instruction in all departments, and their training in nonmajor branches is preponderantly the same as if they pursued a common curriculum. The specialization is attained by shrinking slightly the time to each nonmajor branch and devoting the total increment thus gained to the major. Personnel directors overestimate the significance of this specialization. Seeking a mechanical engineer for quite general work in a factory, for example, they will not consider a civil, mining, or electrical engineering graduate even though he be in the upper percentiles of the class. The evils of departmental specialization lie more in the departmentalized recruiting practices of industry than in the educational results.

All educational programs should aim at the development of the whole man, whether the intellectual part of the curriculum pertains to engineering, law, medicine, social studies, or the humanities. In this respect, all education should

be liberal—that is, directed to the whole free man. It should not be limited to the intellectual realm, but should include the emotional, the volitional, the social, the spiritual, and the physical. The latter qualities are little affected by the particular subject matter in the courses studied and should be given separate consideration. Narrow specialization relative to today's civilization is frequently found in the humanities as well as in the sciences. Any feasible inclusion of the humanities in the engineering curriculum will fall far short of the postcollege reading and reflection requisite of the cultured man. If that small inclusion initiates a taste and an inclination, it will be justified.

Although all of these capacities are educable, most college study is directed solely to the intellectual growth, with some attention paid to the physical. Effective pedagogic processes have not been devised for the other categories of attributes of the whole man. The older British universities do a somewhat better job than do the American; and as a member of the Board of Visitors at the U. S. Naval Academy, the writer has been impressed with the accomplishments of that institution in the development of these personal qualities. To a considerable degree, the nurture of these qualities is subconscious, the product of environmental influences and institutional traditions. The neglect of these disciplines constitutes the chief defect, not only of engineering education, but of all American higher education.

As the author so well points out, postcollege engineering education should have attention. It occurs subsequent to the impressionability of youth and when the outlines of careers are becoming more discernible. It differs fundamentally from graduate study, which is aimed primarily at research methodology and investigation. Inasmuch as it pertains to the profession, perhaps the ECPD might undertake to formulate a suitable program.

The recent reorganization and amplification of the American Society for Engineering Education (formerly "Society for the Promotion of Engineering Education") is significant of an enlarging grasp of engineering education. The appearance of a challenging paper by a practicing engineer, in the *Proceedings* of the oldest of the Founder Societies, is a portent in the same direction. Coupled with the expanding universe of engineering as a result of recent scientific advances, fresh attention to education may be epochally significant to the profession.

N. W. Dougherty,[18] M. ASCE.[18a]—From the practicing engineer's point of view the paper is a good presentation, but it overlooks many of the changes in engineering education that have taken place during the recent past. In the "Introduction" the author suggests that:

> "It [engineering education] has not, however, made comparable progress in keeping abreast of changed conditions that have developed during that period in the pattern and structure of the engineering profession, in the practice of engineering, or in the market for engineering services."

Some of the subjects omitted probably belong to the postgraduate years rather than to college courses. Education is an experience rather than an

[18] Dean of Eng., The Univ. of Tennessee, Knoxville, Tenn.

[18a] Received August 2, 1946.

accumulation of information; it should be inspirational and continuing rather than complete in a four-year program.

All the colleges have been guilty of neglect in their continuing activity after graduation. On the other hand, if they do their in-college instruction in an inspirational way there will be less need of continuing instruction. Why should educated men continue to be spoon-fed after they enter the world of work and application? There are many things the colleges can do to help; but all they can do cannot make educated men if the students do not want to pay the price of education.

Again, quoting the last sentences of the "Introduction": "Every engineering student must work hard to graduate. This probably was the most valuable single thing that he learned in his course." Learning to work is valuable but learning to work effectively is more valuable. If the student has not learned this lesson he probably has not learned much—learned to work with books, the printed page, and rows of books; learned to glean a magazine page and sift the wheat from the chaff; learned to trace cause to effect; learned to respect law but not worship it; and learned to continue in his search for truth.

Prewar Engineering Education.—There is so much difference in the colleges that it is practically impossible to appraise what has, and what has not, been done. When the Engineers' Council for Professional Development (E.C.P.D.) has accredited fifteen different curricula and a host of options, breaking down the fifteen into more restricted fields, one can truthfully say that there has been too much specialization. The author sums up this practice by the following statement:

> "The general objective * * * appeared too often to produce a graduate well versed in theory of, and with an academic knowledge of, practice in one particular branch, and in many instances in one specialty in the field of engineering, thus equipping the graduate to enter such field or specialty on graduation and encouraging him to do so, irrespective of what the market for services in such field might be in five or ten years."

Whenever a student enters any program of study having a definite objective in mind he will try to find employment in that field on graduation although employment is scarce in the field. It is fallacious to believe, however, that the college training in engineering in any field restricts the trainee to employment in that special field. Too many who were trained as mechanical engineers are practicing electrical engineering, or electrical engineers are practicing mechanical engineering or civil engineering, to make the statement that training in a field ties the student to that field. The author draws the conclusion from his premises that either there must be a broad general course, or there must be applications to practice in a single branch or specialty. There are equal possibilities in a broad general course in civil engineering, for example, or specialization in the various fields of civil engineering.

Educators have a notion that they should give enough technology to transmit the attitude, method, and approach to the solution of problems which depend on broad fundamental laws of physics, chemistry, mechanics, and hydraulics. The method and approach can best be acquired by solving actual problems of engineering practice. True, they must be miniature and often

from assumed data to illustrate a principle, but such problems do give an introduction to the engineering approach. To many educators the field of specialization is not so important as the method and the problems.

There is educational value, of course, in sequence courses. The emphasis of engineering schools is not necessarily on specialization but rather on the application of engineering theory to some engineering problem. If the problems come in an area of later employment there will be advantages; and should this not happen no harm has been done.

Except for the idea of "the field of general engineering," the author's course, as outlined, is in general agreement with the recommendations[19,20] of the American Society for Engineering Education—formerly, Society for the Promotion of Engineering Education (S.P.E.E.). Both have the same general objectives, namely, a well-rounded engineer rather than a skilled technician.

The author points to a well-known problem in engineering education:

> "He grappled with problems of a character that he would never meet, or at least would not have to solve, on his own responsibility for from at least twenty to twenty-five years after graduation."

This may be very true; but, if the teaching is to achieve method, attitude, approach, and confidence, no great harm has been done if the student never meets the problem again. For years engineering educators have recognized the problem of dissatisfaction and discouragement of students during the first five to eight years out of college. There is no question that much of the difficulty stems from the failure to use their advanced knowledge in immediate practice. Judgment should come with experience but not all experiences develop judgment. A part of the remedy is to produce understanding employers along with more understanding instruction.

The writer has attended many meetings with practicing engineers where attempts were made to discover "what is wrong with the colleges" and always has heard the statement: "As a graduate he seldom had the ability to express himself adequately or a knowledge of how to acquire this ability."

This seems to be a universal trait. Engineers are about the worst on the list, but it has been discovered in preachers, lawyers, doctors, dentists, and English professors. Good expression seems so easy that all are deceived by it. The practitioner, who cannot express himself, tends to blame some professor for his lack of proficiency. Good English is "a gift of the gods" and the result of real effort to acquire ability. Much of its lack is the result of having nothing to say or of having very hazy ideas. Good expression and clear thinking "go hand-in-hand."

The analogy of the steel mill is a little overdrawn but it does emphasize a well-known educational principle: Men are not materials, and educational experience will not make them mainsprings or auto bodies, but it may make it possible for them to be either. All men act as individuals; some of them are intelligent enough and self-reliant enough to succeed, whereas others are per-

[19] "Aims and Scope of Engineering Curricula," *Proceedings*, S.P.E.E., Vol. XLVII, 1940, p. 555.

[20] "Report of the Committee on Engineering Education After the War," *Proceedings*, S.P.E.E., Vol. LII, 1945, p. 36.

fectly willing to follow the leader. No educational process has been developed to make all of them become leaders.

The Engineering Profession.—The author's study of the engineering profession is thought provoking; it points a way to a better understanding of the task of the colleges. More studies of this kind will be very helpful. What changes of definition have occurred during the forty years of census reporting? From where have the additions come? The author's description of the engineer is probably more nearly true to type than Saint Luke's definition of the Athenians: "That they spend their time in nothing except to hear and tell some new thing." Quoting the paper (see heading, "The Engineering Profession: General Traits and Characteristics of Engineers"):

> "On the credit side it may be stated that engineers in general are resourceful, hardworking, ingenious, and persistent, and have a great devotion to their work. On the debit side they are extremely narrow in their interests and outlook, and highly individualistic."

When engineering is considered "as a heterogeneous rather than a homogeneous profession" engineers are likely to fall into error when they attempt too much generalization regarding the characteristics of their profession; yet the writer believes that Mr. Baker has done well in the foregoing two sentences. By training, and no doubt by birth, the engineer is individualistic. He is not the "back patter" and the "glad hander" of some other professions. Part of that is due to his training and part to his very nature; his disposition caused him to study and begin the practice of engineering. Considerable change will be required to make over the introvert who goes into engineering into the extrovert salesman who disposes of the products of the assembly line. As a matter of fact both types are needed in industry and both types can profit by training in science and technology.

Table 3 is very interesting. A census reported in 1966 will probably raise the percentage of practitioners who enter engineering through the four-year college course. Engineering is now where law and medicine were at the beginning of the present century. Probably it should not aspire to the high percentage of college practitioners now required of law and medicine. The field is far more varied and gives much more opportunity for great variations in innate talent.

Fig. 1, giving a cross section of the engineering profession, is more than interesting. If it is at all to scale, it points to an evolution which is far more revolutionary than the preceding data suggest. The ordinate probably bears some relation to age or time spent in the profession. The author would do the profession a genuine service if he tried to make a similar diagram to scale, using professional age as the ordinate.

Postwar Engineering Education.—The changes suggested under this heading are those usually made by engineering educators as well as by those who practice engineering. Future changes will not be as drastic as those suggested because the curricula are rarely as specialized as the author assumes. More emphasis will be placed on the professional aspects of engineering—the broad bases of engineering practice—and less emphasis will be placed on specialized

skills. For example, the following quotation expresses many of the ideas of the author as he describes his postwar engineering education:

> "For the scientific-technological stem, acquirement by the student of mastery of the basic principles, assumptions, empiricisms, and codes of practice which constitute the subject matter of engineering study, accompanied by acquisition of the ability to apply them to problems of practice which constitute the engineering method.
>
> "For the humanistic-social stem, development of the ability to read, write and speak the English language effectively; to understand, analyze, and express the essentials of an economic, social, or humanistic situation or problem; and to appreciate its implications and relationship to the life and work of an engineer. There is also a goal of development of an adequate concept of the duties of citizenship in a democratic society; and acquaintanceship with the enduring ideas and aspirations which men have evolved as guides to ethical and moral values; and an appreciation of cultural interests lying outside the field of engineering."[20]

The engineering profession owes a debt of gratitude to the author for this thought-provoking paper. For their part, the colleges have already stated what they think should be done in engineering education after the war.[20]

Members of the Society should see to it that civil engineering educators do not fall into some of the snares that have obstructed their paths in the early development of education for the civil engineer.

H. A. WAGNER,[21] ESQ.[21a]—Like Mr. Baker, the writer has become convinced that the entire plan of engineering education should be revamped to meet present problems. The schools should perform three important functions: Train engineers in the scientific approach to the solution of problems; give them a foundation in subjects that are fundamental in business and industry; and provide a broader scope of instruction in subjects that cover the entire field of engineering, insuring a thorough foundation in the basic fundamentals underlying all branches of engineering.

These ends cannot be achieved by crowding additional subjects and disciplines into the present four-year course. The training recommended by Mr. Baker can be given only by eliminating from the curricula certain courses that develop elementary skills and proficiency in the use of scientific tools and instruments, and by deferring to postgraduate study those courses in advanced theory and practice, highly specialized, which now monopolize the time of students in their junior and senior years. The writer concurs in the recommendation that the schools develop strong, postgraduate extension courses that will enable engineers to specialize after they have discovered their own aptitudes and opportunities—after they have learned from experience the conditions that govern practice and the market for engineering services.

To the writer, as to Mr. Baker, this seems the auspicious time to consider drastic revision of engineering education, while curricula are still plastic due to the necessity of adapting engineering training to the exigencies of the war and its aftermath. Mr. Baker is not at all didactic in his specific recommendations for curriculum revision. He is definite and urgent in his plea that the

[21] Cons. Mining and Metallurgical Engr., Chicago, Ill.

[21a] Received August 5, 1946.

schools and the societies conduct an extensive and thorough investigation into the composition and structure of the engineering profession, the conditions that govern and prevail in actual practice, and the market for engineering services—investigation that will provide authoritative guidance for those who may be charged with the task of revising curricula.

The very crux of the entire problem is graphically represented in Fig. 1, depicting the composition and structure of the profession. No one can appraise current curricula intelligently, much less undertake to revise them, until there is a clear understanding of these factors. A thorough investigation of the composition and structure of the engineering profession is an indispensable preliminary to curriculum revision.

After such an investigation has been made, engineering educators can properly evaluate Mr. Baker's specific recommendations for curriculum changes. It may be, however, that certain evolutionary processes, already apparent, will take the situation out of the hands of the profession and its educators. Certain developments are in evidence that already indicate a movement toward the three-pronged system of training which evolved in Germany before World War I.

This system includes three types of training. There is, first, the "Gewerbeschule," which corresponds roughly to the "technical high schools" in the United States. It equips students to use engineering tools and precision instruments, and develops rudimentary skills which are adjuncts to, rather than a part of, engineering. It is quite probable that these schools will be supplemented by union "trade schools" which, under the sponsorship of white collar unions, will serve the same purpose. They will offer only "pre-engineering" instruction.

Then there is the "Technische Mittelschule," which, in its curriculum and discipline, is equipped to prepare students to assume duties more closely related to, or actually a part of, professional engineering. This is done under the close supervision of those who are prepared to direct or perform creative work, requiring exercise of independent judgment. These intermediate schools will produce the indispensable "routineers."

A third type of school produces the fully professional engineer. This is the "Technische Hochschule," which confers state diplomas and doctors' degrees in engineering. Its graduates are masters of the fundamentals in all branches of engineering when they receive their diplomas; they are specialists, when they achieve a doctor's degree.

The present system of engineering education in America is expected to serve the needs of all these groups. Students who will spend their lives in the "subprofessional" bracket, performing only mechanical operations that are adjuncts to, rather than a part of, engineering, and those who will become indispensable routineers, pursue the same studies, submit to the same discipline that develops engineers capable of creative work, involving exercise of independent judgment. If all these groups are considered members of the engineering profession, the schools must continue to serve them all.

This is the problem posed by Fig. 1—a problem suggested, however, rather than stated. Mr. Baker has not the temerity to do what the engineering profession has neglected to do in the past fifty years—define its own scope.

There are four important segments in Fig. 1. It is fruitless to investigate the indicated composition and structure of the engineering profession, unless such an investigation leads to a clear decision, a resolution of the enigma, "What is the engineering profession?" and a question that has long been evaded: "What is its scope?" Eventually engineers must decide "Who are members of that profession?" What is learned from this investigation should enable the engineer to define and delimit the profession; to draw clear lines of demarcation between real professional engineering and the fringe of arts, crafts, and trades which now blend indistinguishably with it.

The profession must do this because it must decide how many of the segments represented in Fig. 1 require genuine engineering education. In accordance with that decision, the responsibility of the schools will be limited and curriculum revision must be controlled. Schools of engineering must know who shall be educated and for what capacities and functions students are to be "processed" before they can plan curricula intelligently.

Mr. Baker has stated plainly that the schools and the professional societies should investigate the composition and structure of the profession. He is not to be censured for withholding his own opinion as to the scope of professional engineering. He intimates that those who have "left engineering" are no longer members of the profession, by the device of a broken line, marking the outer boundary of that segment in Fig. 1. If he had felt strongly that those engaged in "subprofessional" work were not members of the profession, he might easily have suggested it by a similar device—a heavy line of separation or some other graphic representation of detachment. Mr. Baker did not allow himself to be diverted from his major thesis by a discussion of this highly controversial issue. He did unreservedly recommend clarification of the point, urging educators and societies to investigate the composition and structure of the profession.

There are many conceptions of "professional status" and of the "scope of the profession" and each has a substantial number of stout adherents. First, there are those who believe that a degree in engineering is unimpeachable evidence of professional status, of membership in the profession. Second, there are those who believe that, regardless of academic qualifications, performance of engineering duties is *prima facie* evidence of membership in the profession. Third, there is the contention of registered engineers that the engineering profession consists exclusively of those who, by registration, have a statutory right to use the title, "professional engineer." Another faction, more realistic, discounts the statutory restriction of the title as a legal technicality; and, in support of this attitude, cites the vast numbers of eminent engineers who, although qualified, have never chosen to register because the oddly constructed license laws in the United States make it unnecessary to do so. Many argue that by membership grades the engineering societies describe the extent of real professional engineering and designate "professional status." Most prevalent of all, perhaps, is the notion that professionalism is not an absolute, sharply definable state; it is held that there are gradations, and that the measure of "professionalism" is the degree of eminence.

Not one of these notions provides guidance for educators if they attempt to put in effect the curriculum changes recommended by Mr. Baker. The schools have attempted to reconcile all these ideas, and to serve the needs of a profession that is practically undefined.

The theory is advanced that those who eventually enter the segment in Fig. 1 inscribed as "Number engaged in technical engineering work primarily," and that designated "Number engaged in administrative and executive work primarily," must earn their spurs in "subprofessional work." If the profession were to embrace this theory, the schools, regarding the subprofessional group as members of the profession, must continue to offer courses that prepare potentially professional engineers for work involving tracing, drafting, and the use of precision instruments. Graduates in the subprofessional segment will work side by side with men whose training for such duties has been entirely practical, or has been obtained in technical high schools and trade schools.

If this group is not to be regarded as a part of the profession the time now allotted to such courses can be devoted to engineering fundamentals.

Other professions, sharply differentiating professional from nonprofessional practice, train technicians apart from future professionals. Bacteriologists, nurses, laboratory technicians, and other affiliates of medicine prepare for their jobs by taking courses much less rigorous than those required for the practice of medicine. It is impossible for bacteriologists, however brilliant their contribution to medical science, to qualify as physicians without taking the courses required for a doctor's degree in medicine. The line of demarcation between "professionals" and "nonprofessionals" is sharply drawn in licensure; the training of professionals and technicians is as plainly differentiated.

If and when the engineering profession sharply defines its scope, sets up standards of membership as clear and inflexible as those statutorily established by the profession of medicine, engineering education can follow medicine's example also in eliminating courses that merely develop skills used in preprofessional capacities. When this happens, the pre-engineering group, indicated in Fig. 1 as "subprofessional," will be quite definitely excluded from the profession, and it will no longer be the responsibility of the engineering schools. Other types of schools will prepare "subprofessionals" for pre-engineering functions. Whether this group will be excluded, or will secede, is a matter about which the writer will not make any predictions.

If this change comes, it may not be on the initiative of the profession. Professional status is now being defined by governmental agencies—not one, but many agencies. The Civil Service Commission, in its classification of nonprofessional, subprofessional, preprofessional, and professional employees is assuming, but not usurping, a prerogative which engineering has never exercised, drawing distinctions as it sees fit between professional and nonprofessional engineers. The United States Employment Service is preparing a series of booklets for the guidance of its placement counselors, describing the characteristics, qualifications, and duties of engineers serving in professional capacities. The Fair Labor Standards Act (F.L.S.A.) Administrator, after futile efforts to distinguish "professional status" by academic definition, finally resorted to the rule of thumb that, for the purposes of F.L.S.A., any engineer earning less than

$200 a month in base pay is nonprofessional. Case by case, the National Labor Relations Board (N.L.R.B.) distinguishes "professional" from "nonprofessional" engineers. Nothing has as yet crystallized from its many decisions definite enough to be called a line of demarcation; but a pattern is slowly emerging. The Board's line of reasoning can be traced in the following interesting decisions: Aluminum Company of America (61 N.L.R.B. 180); Bethlehem Steel Company, Shipbuilding Division (Case No. 20-R-1256); Gielow, Incorporated (Case No. 2-R-5290); LaClede Steel Company (Case No. 14-R-1148); Neches Butane Products Company (Case No. 16-R-1159); Packard Motor Car Company (Case No. 7-R-1809); Stone and Webster Engineering Corporation (Case No. 1-R-2139); and Curtiss-Wright Corporation (Case No. 9-R-1738). The Bethlehem Steel Company case is particularly interesting.

Another and even more potent force is the appeal of the labor unions who argue that "professionalism" is a fetish, and who assert that unions can do more to improve the social and economic status of subprofessional (and professional) engineers than can any professional or technical society. They scoff at the tradition that engineers are individualists and cannot advance if any restraints are imposed on professional freedom. They insist that strong, collective action insures better pay, improved working conditions, and satisfactory recognition; and point to the relatively high wages of skilled and unskilled labor, in comparison with subprofessional, or even professional, salary schedules, to support the argument. They discount the theory that engineers advance more rapidly when they are free to exercise individual initiative, capitalize on native ability and aptitudes, and bargain directly with the employer.

In any investigation of the composition and structure of the engineering profession these conflicting theories should be tested. If engineering is thought of as a pyramidal structure (and Fig. 1 indicates that it is) there is less room at the top than there is in trades, which are more like cubes made up of a myriad of almost identical units. The engineering pyramid is irregularly stratified; and the individuals of which it is made differ greatly in ability, training, initiative, industry, and those temperamental qualities which so potently affect careers. There is also no inflexible relation between these good qualities and advancement from grade to grade. Opportunities—external factors—influence the progress of individuals from subprofessional to exalted technical rank. A large number of engineering graduates, regardless of their qualities, are destined to remain routineers simply because there is not enough room at the top for all of them.

Some engineers are beginning to be cynical about "professional status." Despite their degrees, graduates spend years in a kind of no-man's-land—not members of any trade, yet denied professional status by their own license laws. Appealing to engineers, disowned by the profession, the unions have convinced a great many of them that "professional status" is a myth, and "professional ethics" a fetish.

There are more practical objections which the unions find it harder to dismiss. Engineers have become identified with too many different unions. They change jobs frequently, and a new position may entail a complete change in union affiliation. It is not easy, therefore, to convince engineers that it is

worthwhile to build a strong union—to pay substantial dues, to demand clauses in union contracts which strengthen the union itself and fight for these more resolutely than for clauses which insure direct and immediate benefits to the members. Although the craft and industrial unions are still absorbing subprofessional engineers (and even preprofessional and professional engineers), the major unions have recognized the disadvantage of this fragmentation, and they are building "white collar unions" in which, apparently, they hope to integrate clerical and "professional" workers.

Those who hope to retain these subprofessionals and routineers as members of the profession, must study the structure of the profession. If the chances of reaching the top are limited, some way must be discovered to offset this disadvantage and to insure pay for the subprofessionals commensurate with their training and services. Their salary schedules must bear such relation to wages of skilled and unskilled labor that they will not find it necessary to identify themselves with these groups in order to protect their interests.

On the other hand, if the profession were to renounce its claim to subprofessionals, eliminating from the engineering curriculum those courses which qualify students to perform subprofessional duties, it must be with full understanding of the effect of that decision.

First, the unions can be expected to set up "trade schools" and systems of apprenticeship such as they now maintain for operating engineers, for plumbers and decorators, and other trades. There will be a corresponding decrease in the number of students who enter engineering schools. Engineers cannot expect to continue half trade and half semiprofessional. If a large proportion of subprofessionals obtain their training in technical high schools and union trade schools, then by collective action establish high rates of pay, and through seniority clauses insure continuity of employment and automatic promotions, four-year courses leading to similar jobs will be less and less attractive.

Second, if the unions, rather than the professions, decide what constitutes "professional status"—if they draw the engineer's boundary lines for him—it may not be subprofessionals alone who will be absorbed. Just above the subprofessionals who perform mechanical operations subsidiary to, but not a part of, engineering, there are the "preprofessionals." These men, under close supervision, implement the plans of professional engineers. They need greater knowledge of engineering fundamentals; they participate intelligently in creative work; but they do not exercise independent judgment and, as engineering becomes more and more standardized, their jobs are correspondingly more and more routine. There is no assurance that these men will not also secede from the profession. If the unions gain control of "preprofessional" as well as subprofessional engineers, the usual routes by which graduates progress to "responsible charge" will be closed to those who are reluctant to join unions. Perhaps some institution, like the intermediate schools in Germany, will develop to equip "preprofessionals" with the knowledge of engineering fundamentals required in these routine jobs, like the short courses offered in the United States during World War II.

Eventually there will be left to the engineering schools only those students who have demonstrated a capacity for creative work or administrative service.

Relatively few institutions of higher learning would be needed to "process" the number of engineers of this category that industry can absorb. It would be necessary for industry to accept the product of these schools as trainees, or understudies, instead of developing them gradually in subprofessional and preprofessional capacities.

The effect of this shrinkage upon engineering societies and upon employer-employee relations must be given careful consideration. If, by professional definition or by their own secession, a preponderance of subprofessional and preprofessional engineers are definitely separated from the profession, the structure of the societies will be changed as much as the structure of the profession. Engineers will not support both unions and societies. So long as they have a tenuous claim on professional status—indicated by such evasive designations as "preprofessional" engineers, or "engineers-in-training" or "engineering aides" —they may remain loyal. If through any act of the profession, or on their own volition, they become definitely recognized as nonmembers—outsiders—they can hardly be expected to remain in the societies.

Certainly the relation between engineer-employers and engineer-employees will be changed if a great majority of employee engineers in subprofessional and preprofessional ranks organize in the pattern of, and identify themselves with, skilled and unskilled labor. These organizations will be in a position to control the "processing" of men who will join their ranks just as they now control apprenticeship and training for the trades.

To the writer it seems Mr. Baker's plea for a thorough investigation, by engineering educators and technical societies, of the composition and structure of the engineering profession should be heeded. It should be acted upon before other agencies have decided, for the profession, that crucial question—"Who shall be members of the engineering profession?" This is the essence of professional freedom. It is a right, cherished by all other professions, which is given to a professional man in exchange for the service he renders to society, in order that he may not be impeded in such service. If it is not too late, the writer would strongly advise the profession to follow Mr. Baker's urgent recommendation. The present composition and structure of the engineering profession should be investigated thoroughly, and standards of qualification for membership should be redefined—new boundary lines for regulating entrance to the profession. Then let educators plan to "process" engineers so defined in engineering schools for utmost usefulness to industry and society.

M. E. McIver,[22] Esq.[22a]—Mr. Baker has urged that engineering educators and the technical societies examine carefully: (1) The composition and structure of the engineering profession, (2) the conditions encountered by engineering graduates in actual practice of engineering, and (3) the factors that govern the marketability of their services. He has recommended that the societies utilize the vast amount of authentic source material accumulated in their own files—the case histories of members, and the records of the employment service jointly sponsored by the societies.

---

[22] National Secretary, Am. Assn. of Engrs., Chicago, Ill.

[22a] Received August 6, 1946.

Even now the Engineers Joint Council (EJC) is engaged in investigations along the lines suggested by Mr. Baker. The EJC, however, seems to be relying on the questionnaire method of obtaining data, instead of using the treasure-trove of facts stored in the files of the societies represented on the EJC.

American Association of Engineers (AAE) has tested the technique of investigation recommended by Mr. Baker and found it far more satisfactory than the questionnaire method of collecting data pertinent to the social and economic problems of the profession.

The disadvantages of the questionnaire method are:

1. Abuse of the device by commercial agencies has reduced its effectiveness. The promotional purpose of such agencies is thinly disguised by the pseudo-scientific veneer of their inquiries;
2. Employed engineers, unless they have been markedly successful, are reluctant to chronicle their careers for their societies and alma maters;
3. In answering questionnaires, employers and employees are inclined to report facts in a manner that reflects credit on themselves and their organizations; and
4. Sheer negligence and procrastination account, in part, for the necessity of repeated follow-ups to secure an adequate volume of replies.

For all these reasons, AAE has found the questionnaire method a slow, costly, energy-consuming process of collecting information. Its committees are inclined, moreover, to discount the accuracy of voluntary replies to inquiries that individuals may consider unwarrantably personal or intrusive. Whenever this device is necessary the questions are composed in such a way that the individual receiving the questionnaire will be intrigued or "needled" into replying.

Sources of Authentic Data.—The AAE employment service is its most dependable source of information concerning the social and economic status of engineers, the conditions that prevail in actual practice, and the market for engineering services. It is also the most vital contact with employers of engineers. Case histories of applicants, placement records, job orders, etc., are assembled by the employment service as routine; and AAE committees thus have more accurate information than could be obtained by a questionnaire. All their energies can be expended on organization and interpretation of data, because this system eliminates the arduous preliminary effort of collecting information by questionnaire. Fundamental steps in the procedure recommended by Mr. Baker are demonstrated in an AAE study published in June, 1946.[23]

Corroboration of Evidence in Mr. Baker's Paper.—Nowhere in Mr. Baker's paper is there any suggestion of authoritarianism. The evidence he has introduced in support of his convictions for the revision of engineering curricula is offered, not as conclusive proof of these recommendations, but to demonstrate the need of a more thorough investigation of the conditions indicated by these data and to suggest desirable lines of investigation. Various studies made by AAE committees support Mr. Baker's basic recommendations and corroborate

[23] "Before and After VJ-Day," *Professional Engineer*, June, 1946, p. 5.

evidence that he has cited.[24,25,26,27] Like Mr. Baker, the writer is prepared to demonstrate the absolute truth of these propositions. Nevertheless, the findings from investigations, although limited in scope, are interestingly parallel to, or supplement, the facts and findings recorded by the author.

Overspecialization.—From engineers and from their employers, the writer has gained the impression that engineering education is overspecialized. Employees exhibit a certain timidity that is attributable to overspecialization. It engenders a reluctance to accept engineering assignments outside a very restricted technical area in which they have specialized.

Electrical engineers, for example, thoroughly grounded in the theory of fractional horsepower motors, consider themselves competent to handle only the specialized phase of design of household appliances. They refuse to accept responsibility in the production of these devices because they lack knowledge of mechanical engineering principles, or the metallurgy involved in a production assignment.

In their job orders, manufacturers of electrical appliances manifest the same skepticism; they demand "mechanical engineers with experience in electrical lines," instead of electrical engineers for production jobs. Employers have greater confidence in specialized experience than in specialized education.

These are impressions—some exploratory work has been done by AAE to test the theory, but, as yet, nothing is conclusive. A study of the effect of specialization on the careers of ex-service men reveals that they fall into four definable units.

First, there are the men whose technological education was derived exclusively from special courses given by some branch of the armed forces, and who expect to continue this line of work in civilian employment. Second, there are the men whose academic training was interrupted by induction into the armed forces. The third unit is made up of men who, although they had completed four-year courses and received degrees in some branch of engineering, were inducted before they had sufficient experience to qualify for a position in responsible charge of work. Finally, there is the group that, having attained full professional stature, entered the armed forces as technical specialists.

Taking care to secure evidence from all four types of ex-service men, AAE asks the subjects to answer a questionnaire, which supplements their experience records, and which is obviously designed to aid the placement counselors in their special problems. Unobtrusively, a series of questions is woven into this questionnaire which, considered in relation to each other and to the applicant's complete vocational history, will provide a body of interesting facts concerning the effect of academic specialization on engineering careers.

The questionnaire asks, not only about the degree of specialization, but about the circumstances that prompted the candidate to concentrate upon a special phase of engineering in college. The relation of his subsequent experience to this specialization is investigated, measuring the well-known effect of

[24] *Professional Engineer*, November, 1940, p. 35.

[25] "The Engineer as His Employer Sees Him," *ibid.*, February, 1928, p. 5.

[26] "Nation-Wide Survey of Engineering Civic Status," *ibid.*, December, 1939, p. 2.

[27] "Survey of the Market for Engineering Services," *ibid.*, November, 1940, p. 3.

the economic depression on candidates for employment who during the 1930's took any available job, regardless of specialization. The aim is also to measure the effect of rapid technological change during the 1940's upon the employability of highly specialized graduates who have not been engaged in engineering during the war. Finally, the objective is to ascertain the effect of wartime experience on the career plans of men who, if the war had not interrupted those plans, would have followed a very narrow path of specialization.

This line of investigation is one that the EJC might profitably pursue, expanding the exploratory study begun by AAE into a thorough, full-scale investigation. For obvious reasons, ex-service men (representing all four groups) are the best possible subjects for a study of specialization. Questions that concern the subject personally are emphasized in the AAE questionnaire: Rapid changes in technology during his period of military service that may, in some degree, have outmoded his academic training or experience; "GI" rights to reemployment and educational subsidies; salary modifications that take into account his maturity and wartime experience. This illustrates the technique for overcoming the inertia, reluctance, or actual hostility that makes the questionnaire method generally unsatisfactory.

Some of the early replies to the AAE questionnaire support incidental findings of other committees. Several of the subjects have shown that they were strongly influenced by parents or faculty advisers in choosing specialties, and that they are about to "write off" that theoretical asset as a loss. They ascribe various reasons for changing career plans. Some of them find a particular line of specialization not as lucrative as they had hoped; some report the field overcrowded; others complain that the specialty entails confining work; a few have come to regard the line of work for which they prepared as "socially unimportant" in comparison with other projects with which they were associated in wartime.

The most casual examination of these documents reveals that these men confirm the opinions of employers of engineers that were reported by AAE in 1928.[25] Employers intimated that, under a system of academic specialization, young engineers have made this crucial choice when they lacked maturity to insure good judgment; they made it, moreover, without adequate knowledge of the conditions governing practice, or of the market for such services. Employers declared, rather than intimated, that the product of engineering schools was precocious in its knowledge of advanced engineering theory but deficient in appreciation of the business side of engineering, and that overspecialization was achieved at the cost of essential training in engineering economics.

At the same time, employers noted that a considerable proportion of students' time was spent in acquiring elementary skills and learning to use engineering or scientific tools and instruments; this, also, was at the expense of sound training in fundamentals throughout the broad field of engineering. This condition, employers reported, accounted for the low rates of compensation received by a large segment of engineering graduates, who, using only these elementary skills and tools, compete with subprofessionals whose competence is based entirely on practical rather than academic training. The 1940 "Survey of the Market for Engineering Services" to which Mr. Baker has referred,[24]

reports evidence in "job orders" which corroborates the testimony of employers quoted in the 1928 report.[25]

It is only undergraduate specialization, obtained at the expense of broader training in engineering fundamentals, that is decried by employers. Engineers need these advanced courses after they have discovered their shortcomings in actual practice, or when opportunity for advancement depends on additional study. They are insatiable in their quest for scientific knowledge; it is this fact that explains the multiplicity of technical societies, and the sectional and cross-sectional pattern of organization within the societies. Mr. Baker might have argued that engineers, in this pattern of organization, demonstrate a potential responsiveness to systematic extension courses that may be offered by the schools. Undergraduate specialization is highly opportunistic. Postgraduate specialization is purposeful. If the schools offered well-planned, practical extension courses, engineers who have already embarked upon careers, who know the market for certain specialties, and who recognize their own limitations, should certainly be expected to avail themselves of such facilities quite as eagerly as they now seek advanced training in theory and practice from technical society meetings and technical society journals.

Mr. Baker's Analysis of the Engineering Profession.—No psychological tests and no scientific methods have been devised by AAE, that enable its committees to report authoritatively the mental quirks of engineers, or to trace tendencies toward an engineering education. The author has accurately reported certain general indications noted by AAE committees. A synthesis of employers' opinions, as expressed in job orders, presented in the 1940 survey[24] demonstrated that employers believe engineers' principal weaknesses are: Blindness to intangible phases of their work; inarticulateness; inability to sell their own capabilities; and an indifference to, or lack of, understanding of the "profit motive" involved in engineering projects. The same criticism was found in the earlier 1928 survey,[25] and to some extent in the 1939 survey.[26]

The 1946 study[23] leads one to conclude that, since 1941, there has been a marked increase in employer demand for engineers who can "count the cost." It seems to be an outgrowth of the change-over from "cost-plus" systems of production of war goods to normal, competitive methods of producing civilian goods. There are other reasons.

Fearing inflation, and confronted by increased pay schedules for skilled and unskilled labor, business needs engineers who can get maximum returns from this more expensive labor force and so hold down unit costs of production, thereby averting an increase in prices of finished products. Dollar-conscious engineers are needed also to keep engineering costs in line with economy programs designed to offset high taxes.

Manufacturers need engineering executives who can understand and insure conformity with all manner of government regulations—for example, those of the Securities Exchange Commission, the Internal Revenue Department, and others which impose highly technical rules and require complex and meticulous records in which technological operations are a factor.

Finally, business needs expert engineering guidance in order to arrive at, and conform to, satisfactory collective bargaining contracts. Labor unions are

represented by experts in negotiations for contracts and in grievance procedures; the employer must have business administration experts who understand engineering, or engineers who grasp the fundamentals of business administration in order to deal satisfactorily with the unions in setting up job classifications and specifications, wage and incentive systems, etc., and in living up to the very letter of agreements with the unions.

Mr. Baker omitted only one of the insistent demands made by employers in reporting the findings of the 1940 survey.[24] This was the growing demand for engineers of "pleasing appearance." Perhaps the author felt that this particular problem is the responsibility of the individual, not the schools. A number of AAE committees have suggested (and the point was made in the 1940 survey) that the schools may have rather overemphasized "ruggedness." Some misapprehension may have caused the students to confuse virility with untidiness. At one time, a committee requested the writer to check, casually, the rather prideful references published in engineering school journals to engineering students as "campus roughnecks." There was some corroboration of that theory, but it was not sufficient to warrant examining a great number of these journals.

For the 1940 survey, however, employer specifications were checked closely as to "appearance" of candidates. Of the job orders filed with AAE between 1927 and 1929, 1.6% stated that appearance would be a factor in evaluating candidates for jobs, and in 1939 and 1940, 13.6% of the orders made a similar requirement.

Census Bureau Statistics Quoted by Mr. Baker.—Mr. Baker has reproduced three interesting tables compiled by the U. S. Census Bureau, showing the distribution of technical engineers on the basis of educational attainments, age, and salaries. The author does not argue that these tables are necessarily representative of the entire profession. He does suggest that they are interesting, as they pertain to a substantial segment of cross section of the profession.

The 1940 survey included an analysis of data concerning 573 engineers who, at that time, made up the "active file" of registrants in the AAE employment service.[28] Segregating those qualified for responsible charge of work in each category, the distribution was as shown in Table 10. These groups were analyzed from the standpoint of educational qualifications, age, and salary ratings; but the AAE experiments are not exactly parallel to those made by the Census Bureau and reported by the author. For instance, AAE dealt exclusively with "employee" engineers, whereas the Census Bureau group included an unstated percentage of engineers engaged in private practice. In analyzing educational attainments and salary levels the AAE procedure was not precisely that of the Census Bureau. Certain findings are reported in Table 10 only because they bear a striking similarity to those of the Census Bureau.

For example, Table 3(*a*) can be compared with Table 10, which shows the distribution of the 573 engineers whose registration cards were the basis of the 1940 study.[29]

---

[28] *Professional Engineer*, November, 1940, p. 26.
[29] *Ibid.*, Group II, p. 29.

For civil and electrical engineers, the data in Table 10(*a*) are not greatly at variance with those in Table 3(*a*). A smaller percentage of the mechanical engineers, included in the Table 10(*a*) study, held degrees than did the corresponding group analyzed by the Census Bureau.

TABLE 10.—SIGNIFICANT FINDINGS IN THE 1940 SURVEY OF THE MARKET FOR ENGINEERING SERVICES

Description	Civil and construction	Electrical	Mechanical and industrial	Total
Number of records	249	88	236	573
Responsible charge of work (%)	59	60	65	62
(*a*) EDUCATIONAL ATTAINMENTS (PERCENTAGE OF TOTAL NUMBER)				
Degree in engineering	61.0	67.0	36.0	....
Some technical education	22.5	21.5	33.0	....
Trade and correspondence school	6.0	4.5	8.5	....
High school only	10.0	6.0	22.5	....
(*b*) AGE DISTRIBUTION (PERCENTAGE OF TOTAL NUMBER)				
Younger than 31 years	36.5	52.0	38.5	39.0
Younger than 40 years	69.5	81.0	69.0	70.0
Older than 45 years	18.5	12.5	19.0	18.0

Considering all registrants in Table 10 (573), and comparing their educational attainments with the Census Bureau data for "all technical engineers" (Table 3(*a*), Col. 6), the AAE group corresponds very closely to that studied by the Census Bureau. Of the 573 engineers, 52% were graduates, and an additional 26% had one or more years of technical school training (not including trade or correspondence school work). The total 78% practically tallies with the Census Bureau total of 78.8%, adding lines 1 and 2 of Col. 6 in Table 3(*a*).

Table 3(*b*) indicates that 36.2% of "all technical engineers" (Col. 6, line 11) are older than 45. Only 18% of the engineers covered by the AAE study were in that age group. This is natural, because all the 573 engineers were applicants for employment, whereas, the Census Bureau studied both employee engineers and those engaged in private practice. Only 12% of the men placed by the AAE service in 1940 were older than 45.[30] In analyzing job orders it was found that one third of all employers filing such orders made a preliminary requirement as to age, and that only 1% of them expressed a willingness to hire men older than 45.

Although age restrictions were relaxed during World War II, the 1946 study[23] indicates that they have "tightened up" rapidly since the end of the war. The report of placements for 1939–1940 given in the 1940 survey,[30] shows an interesting resemblance to data quoted in Table 3(*c*) (Col. 6, lines 16 to 23). The annual salary ratings of engineers can be studied by segregating the entire sample into three groups of equal numbers; thus, one third of the engineers

[30] "Survey of the Market for Engineering Services," *Professional Engineer*, November, 1940, p. 48.

placed by AAE in 1939 and 1940 received annual salaries of: Less than $1,500; $1,500 to $2,400; and $2,400 to $4,200, respectively.

The 1940 study cites only the rate at which these men were placed, not the actual earnings for the year. The Census Bureau quoted actual and total earnings for 1939, which may have accrued from less than a year of employment, some of the men reported having worked only intermittently but at monthly rates higher than total earnings for the year seem to indicate. Although no real parallelism exists, a comparison of the AAE data with the Census Bureau's report that 21% of all technical engineers earned less than $1,600 in 1939, whereas another 29.8% earned between $1,600 and $2,500, produces no startling contradictions. In Col. 6, Table 3(*c*), adding lines 16 to 19, inclusive, indicates that 21.1% of all technical engineers earned less than $1,600.

Professional Registration.—Among the 573 engineers whose experience records were analyzed in Table 10, only forty four were registered as professional engineers, that is, 7½%. Of the 573, there were 355 who had experience in design or higher grades of practice that should have entitled them to register. Of this 355, only 12% were registered. Of twenty-two construction superintendents, only seven were registered. Only two of nineteen electrical engineers with experience in responsible charge were registered; among the mechanicals only one of ten who had served as chief engineers in industry was registered, and he was the only man in this category who had had no academic training. Employers' indifference to registration was evidenced by the fact that only one of several thousand job orders specified that registration was requisite. This is as true in 1946 as it was in 1940. In the 1946 study of job orders[23] only one employer stipulated that candidates must be registered architects, and not one employer asked for a registered engineer.

Summary.—The foregoing comment is presented on the personal responsibility of the writer and does not necessarily reflect the official views of the AAE. The intent has been to report data compiled by AAE that should be useful in the appraisal of the paper by Mr. Baker. As stated, some of the AAE committees have followed the very line of investigation recommended by the author and have used the technique that he suggests to the schools and professional societies. The findings reported by AAE bear a striking resemblance to those that resulted from Mr. Baker's independent investigations; and the inferences from the findings conform rather closely to Mr. Baker's recommendations that:

(1) Undergraduate education should be confined to engineering fundamentals and that it should cover the entire field of engineering rather than a single branch or specialty; and

(2) The schools should assume responsibility for specialization through systematic extension courses as well as postgraduate study in residence.

Scott B. Lilly,[31] M. ASCE.[31a]—This presentation of the views of a consulting engineer on engineering education is most timely. If changes in the curriculum are due, they should be made now while the effects of World War II

[31] Prof., Civ. Eng., and Chairman, Div. of Eng., Swarthmore College, Swarthmore, Pa.

[31a] Received August 12, 1946.

are being felt; while every school is faced with changes in personnel; and while the first-hand experience with teaching skills and techniques gained from the instruction of great numbers of men during the war period is available. The writer agrees with the main thesis of the paper, that engineering education should emphasize fundamental principles, and that postgraduate education should be designed to prepare for a career involving administrative, executive, and managerial activities. There are certain phases of the problem of instruction, however, to which the writer would like to call particular attention.

Essentially, engineering is planning. It consists in meeting and solving problems involved in the design of a particular structure, machine, or business activity, before an investment is made. For its successful practice, the engineer must have a sound background, not only in science, but in experience. In fact, he must have the sound judgment based on experience if he is to create any enterprise that is successful. Under any type of government, investments must satisfy social needs if they are to be justified.

If, then, the great need of the successful engineer is judgment based on experience, why should a man go to college to study engineering? Why not revert to the old apprentice system? Certainly the details of practice can be learned more quickly and more thoroughly on the job than in the classroom. What can a college education give a man that he cannot get on the job? Just this: Unless a man has a theoretical background, he has no way of classifying and evaluating his experience. He has no knowledge of the limitations imposed by nature, no realization that, although a law may hold under certain conditions, abrupt changes may be encountered if its use is extended beyond certain definite limits.

If basic science is to provide this framework, it must be taught so that a greater proportion of students find it usable. Mathematics must be so thoroughly taught and learned that general principles expressed in symbols are readily understood; but these principles must be so thoroughly comprehended that the student realizes that they are useless unless he can get from them the answer to the concrete problem to be solved. The physical sciences must be so thoroughly mastered that a knowledge of theory will make the engineer able to predict the limit of what any approach can be expected to yield, together with a clear idea of the limits of the theory.

The heart of this suggestion is that the amount of material taught should be limited to the ability of the students to comprehend clearly what is presented. Nothing is so useless as a knowledge of facts so poorly digested as to be incapable of use. The author states that too many engineers are "narrow." There is nothing inherent in engineering which makes education in that field "narrow." From the reports that are coming out of the liberal arts colleges, it appears that English, history, political science, and many others of the so-called humanities, are being taught so that the end product is narrow. Quoting from Alfred North Whitehead:[32]

> "In the teaching of science, the art of thought should be taught; namely, the art of forming clear conceptions applying to first-hand experience, the art of divining the general truths which apply, the art of testing divinations, and the art of utilizing general truths by reasoning to more particular cases

[32] "Aims of Education, and Other Essays," by Alfred North Whitehead, p. 81.

> of peculiar importance. Furthermore, a power of scientific exposition is necessary, so that the relevant issues, from a confused mass of ideas, can be stated clearly, with due emphasis on important points."

Engineering is being taught in this way in some engineering schools. A man who has been exposed to this discipline for four years cannot be called "narrow" or uneducated. Every thoughtful teacher in every course should be striving for the objectives stated in the quotation from Mr. Whitehead, whether the subject be mathematics, English, or history.

When he graduates, the student must realize that he has merely learned the vocabulary in college and that he must educate himself on the job. The interest that comes from immediate use, the joy of accomplishment that is experienced when, by reason of study the night before, a man is able to have the right answer at the right moment, gives the urge which may make such study a lifelong habit.

What has been said applies to the technical side of a man's life. Education should prepare a man for his leisure hours as well as his working hours. There is a real adventure involved in the meeting of great minds in books, in making friends with these authors, in the feeling that they are always there on the shelf, where they can be consulted. This type of companionship is waiting for those men who know how to read ideas and not words. It comes to those men who have been under the influence of teachers and friends whose joy is in introducing young men to these pleasures. The teachers who do this must be men endowed with enthusiasm, as well as with knowledge and taste. Is this too much to ask of those who present literature, psychology, and philosophy?

This is not an idle dream. Such teachers can be had if society wants them. Salaries can be made attractive enough to bring able, well-rounded men into teaching. However, men of the type desired crave recognition of the importance of their job even more than they crave money. They must feel that there is wide realization of the great difficulties involved in teaching; of the hours of study necessary; in general, that teaching demands the best that a man has, or can become, if he is to have real success; and that the ends sought justify such efforts.

Such teachers would produce students with a professional attitude. They would be conscious of the fact that they have an obligation to extend the frontiers of knowledge in their own fields. More than that, they would have a knowledge of the impact of their activities on society. They would feel a responsibility toward society; with the background they had received from contact with active minds, with cultivated imaginations, they would be able and willing to engage in public affairs; and the problem of professional recognition for engineers would disappear.

With the help of industry, the teacher can do much to improve his condition. Let him spend some time each summer in industry; let him become familiar with current problems. If he knows these problems, the teacher's salary must rise or he will stay in industry. There is nothing like competition, not only to increase salaries, but to develop the teacher. Teachers must be alert, keen, broadminded, and well informed, if they are to be equal to today's needs.

There is no easy simple solution to this complicated problem. It is not one that can be solved by changing curricula, or studying in college five or six years instead of four. It is quality of instruction that is needed; it is the true professional attitude toward all activities that must be inculcated, that will finally win the battle; and quality in the student must come from quality in the faculty. What can be done to bring into teaching the best minds in the profession? That is the question to be solved. What can the members of the Society do to bring this about?

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

DESIGN LIVE LOADS IN BUILDINGS

Discussion

By D. Lee Narver, and N. N. Freeman

D. Lee Narver,[5] M. ASCE.[5a]—It should be of interest to know that a live-load reduction formula for the design of buildings, developed to accomplish the results described by the author, was proposed to the Committee on Live Loads of the American Standards Association in October, 1939. The proposal went through the "crucible" of criticism of that committee and, after changes, was adopted in the form presented by the author. The proposal was first published in pamphlet form—"Minimum Design Loads in Buildings and Other Structures"—on May 22, 1945, by the American Standards Association (ASA—58.1—1945) and sponsored by the National Bureau of Standards. On October 19, 1945, the material was issued by the U. S. Department of Commerce in pamphlet form.[6]

The object of that committee's work, together with that of committees on other phases of building construction, was to formulate principles that can be used by cities throughout the United States in the development of uniformity of building codes.

In the sentence preceding Eq. 2, the author states that the formula limiting the "design live load" is so developed that, even if the "basic live load" is attained over large areas, design stresses will not be exceeded by more than 30% of their value. It is the writer's thought that this does not tell the full story. Consider the load values given for the Internal Revenue Building (I R Building): Since the maximum $\Delta W_L = \frac{100 + 180}{4.33 \times 100} = 0.645$, the minimum "design live load" is 35.5 lb per sq ft, and the minimum total design load = 180 + 35.5 = 215.5 lb per sq ft. This loading will develop the full design stress. The "basic live load" plus dead load equals 100 + 180 = 280 lb per sq ft, which will produce 130% of the design stress.

Note.—This paper by John W. Dunham was published in April, 1946, *Proceedings*.

[5] (Holmes & Narver), Los Angeles, Calif.

[5a] Received May 24, 1946.

[6] "American Standards Building Code Requirements for Minimum Design Loads in Buildings and Other Structures," *Miscellaneous Publication No. M-179*, U. S. Bureau of Standards, U. S. Dept. of Commerce, 1945.

If steel is the structural material being used, either as rolled shapes or in reinforced concrete, specifications A-7 of the American Society for Testing Materials establish a minimum yield point of 33,000 lb per sq in., or 165% of the allowable design stress of 20,000 lb per sq in. The net effect is 165% − 130% = 35% of the design load of 215.5 lb per sq ft, or 75 lb per sq ft, as the load that must be added to the "basic live load" to develop the minimum assumed yield point. This means an overload capacity of the structure of $\frac{75}{100}$ or 75% of the "basic live load."

The I R Building is an example of office building loading on the heavy side of average; but consider an office building on the light side, whose dead load is 130 lb per sq ft and whose "basic live load" is 50 lb per sq ft.

By the proposed formula (Eq. 1): Maximum $\Delta W_L = 0.83$, or a minimum "design live load" equals 8.5 lb per sq ft. The design load equals 130 + 8.5 = 138.5 lb per sq ft.

Under the same stress assumptions as before, there is 35% of the design load, or 48.5 lb per sq ft that must be added to the "basic live load" to develop the minimum assumed yield point. In this case, the overload capacity of the structure amounts to $\frac{48.5}{50} = 97\%$ of the "basic live load."

Both of these examples show a substantial overload capacity. The yield point of steel used in the foregoing calculations is a required minimum. For several years job tests have been showing the yield point of structural steel to be greater than 40,000 in tension; and in bending it is still higher. Assuming in practice that the yield point will not be less than 40,000 lb per sq in., Eq. 1 will result in an overload capacity of the structure of 150% over the "basic live load" for the I R Building and one of 194% for the example of a building having a dead load of 130 lb per sq ft and a "basic live load" of 50 lb per sq ft.

It is to be noticed that a limitation has been placed on the proposed live-load reduction formulas. It is recommended that assembly occupancies have no live-load reduction. This is a sign of warning that there may be other classes, or particular cases, of loading for which the formulas should not be followed. It is difficult to set up terms of a building code that will always be applicable, and the engineer using the proposed system of reduction should consider carefully whether or not his particular problem falls within the usable limits of the formulas. For example, a specialized storage building, in which full "basic live load" capacity will be attained during known seasonal periods, should have no reduction of live loads.

It appears that the proposed system of arriving at loads for design purposes is closer to giving a true result for the normal run of buildings than the systems now in common use. However, as in the case of all design, the closer the design approaches true conditions, the more important good engineering judgment becomes. It is a step in the right direction toward economic design, and economy is as much a consideration for the engineer as is a safe design.

The proposed system may involve more engineering work than the present methods, but this is a business problem and should not be an obstacle in the way of adoption if the system improves engineering practice and results in economies of construction.

N. N. FREEMAN,[7] ESQ.[7a]—In bringing this important subject to the renewed interest of the engineering profession, this admirable, clearly written paper marks a forward step.

The surveys conducted under the author's supervision seem to have been limited in scope, however, and were not quite correct in procedure. The number of observations was obviously small; and the recurrent use of the expression "* * * at the time it was surveyed * * *"—indicates that most of the data are the result of only one observation made at a certain time and date. It is agreed that the live loads representing furniture and movable property may be fairly constant during a prolonged observation period. Persons, on the other hand, represent a very active live load, and figures may change considerably. There may be rush hours, queues, or conferences, and it would have been advisable if more extensive observations could have been made to determine the maximum actual live loads imposed by people in certain areas of the entire building. Such maximum figures, together with the more constant live loads, would then represent the actual live loads for such a structure.

For the aforementioned reasons, data shown in Tables 1 and 2 represent only actual live loads at a certain time and date, and do not express critical live loads for the buildings in question.

Even if the loads given represent maximum actual live loads, data found in the survey of only two structures cannot be applied as general rules to all office buildings. Only extensive and prolonged investigations into many different types of such structures can supply a basis for an estimate of reasonably correct general design loads which represent actual conditions. Such investigations doubtless require considerable time, but zeal and money invested would be repaid many times over by savings in the design of new structures.

The author's proposal for determining amended design loads rests upon a basic assumption which has not been supported by any experimental evidence. He proposes to allow an occasional overstress in certain parts of the structure. Then he assumes that if this overstress is limited to 30% it is not dangerous and may be ignored in the design.

Whether such an assumption can be accepted without the support of extensive and complete survey data is certainly open to discussion. Such an overstress may reach more than a purely theoretical 30% and could cause a serious reduction in the factor of safety of the entire building. There are certain well-known influences and stresses, however, which only rarely can be included in designs, such as secondary stresses due to stiffness of joints in welded steel frames and in reinforced concrete constructions, uneven settlement of foundations, sudden gusts of wind, and earthquakes. In a proper design such stresses are allowed for by an ample factor of safety, the rate of which is fixed according to local conditions and experience. Even small overstresses at single points may be dangerous, because weakness is introduced into vital construction members and failure may occur in the case of any emergency.

Another obscure point is the assumption of a reduction rate of 0.08% per sq ft of tributary area. No information is given in the paper as to how this

[7] Civ. Engr., Haifa, Palestine.

[7a] Received July 18, 1946.

value was determined; and there is no clue indicating any special argument in the survey data.

It should be the purpose of further surveys to collect ample actual live-load data in many types of buildings over prolonged periods. Only then will it be possible to conduct a proper statistical investigation that would form the basis of recommendations for a new design system approaching actual conditions.

Before such data are available it seems reasonable to make certain reductions in the basic live loads, however, and the value of 50 lb per sq ft, with a corresponding reduction depending on the number of floors supported, should give safe design values. Especially important, also, is the recommendation by the Building Code Committee of the National Bureau of Standards, U. S. Department of Commerce[8] that a load of 2,000 lb be placed on any 2.5 ft of floor space wherever this load on an otherwise unloaded floor would produce greater stresses than the 50 lb per sq ft of distributed load.

With reference to apartment buildings the author reveals no new information and his computations are based upon assumed values, which may, or may not, approach actual conditions. In the design of apartment buildings, also, it would be advisable to collect proper and ample experimental data before any attempt is made to introduce a new method of analysis.

The chapter on warehouses includes some valuable observations; but a limited survey of only eight buildings cannot form a sufficiently broad basis for any general conclusions to be applicable to all warehouses. Additional information showing types of stores involved, movements of ingoing and outgoing consignments, and loading and transport facilities, is needed for a proper classification of data. Certain reductions in design loads seem to be feasible but further surveys are needed before any such assumptions can be made.

Conclusions.—On the basis of the foregoing, three conclusions are possible in a fair appraisal of this paper:

1. Survey data given are incomplete and insufficient to form a reliable basis for any new proposals to reduce design live loads.

2. Overstresses during design can only be permitted if it is desired to reduce the factor of safety accordingly. It should be proved that a limit of 30% is not dangerous, and the author should explain further how the reduction rate of 0.08% per sq ft of tributary floor area was determined.

3. Further extensive surveys should be made to collect sufficient reliable information as a basis for selecting new reduced live loads which approach actual conditions.

---

[8] "Steel Construction," A.I.S.C., New York, N. Y., 1930, p. 52.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RELIEF WELLS FOR DAMS AND LEVEES

Discussion

By Henry C. Barksdale, Willard J. Turnbull, and Glennon Gilboy

Henry C. Barksdale,[7] Assoc. M. ASCE.[7a]—It seems to the writer that Messrs. Middlebrooks and Jervis are to be commended highly for their careful and intensive studies and for the clarity with which the results have been presented. The note of moderation that permeates the whole paper and prefaces the presentation of the theoretical design is particularly gratifying. Experience with the development of ground-water supplies and studies of their characteristics lead the writer to believe that the uniform conditions and homogeneity of the aquifer, which must be assumed to apply any theoretical methods to the solution of such problems, is the exception rather than the rule. Sound engineering judgment is of prime importance in the application of theoretical equations to ground-water conditions. The following comments are intended to be supplementary rather than critical.

Reference is made in the paper to the difficulty of obtaining accurate samples of material at great depths for the determination of the coefficient of permeability. This difficulty is one that has long confronted students of ground-water hydrology. No inexpensive method of obtaining satisfactory samples has been devised. However, the results of pumping tests for determining the coefficient of permeability are being used increasingly. The several methods of determining permeability from pumping tests have been discussed fully by L. K. Wenzel,[8] Assoc. M. ASCE. They depend upon careful observation of the effect of pumping one well upon the water levels in other wells near by. The coefficient of permeability thus determined automatically averages the irregularities of the formation. It is suggested that, in structures of major importance, test wells and pumping tests on them would be a reliable and not unduly expensive method of determining the permeability of the materials beneath the foundation. Some of the test wells conceivably might later be used as relief wells.

Note.—This paper by T. A. Middlebrooks and William H. Jervis was published in June, 1946, *Proceedings*.

[7] Hydr. Engr., U. S. Geological Survey, Trenton, N. J.

[7a] Received June 26, 1946.

[8] "Methods for Determining Permeability of Water-Bearing Materials," by L. K. Wenzel, *Water Supply Paper No. 887*, U.S.G.S., Govt. Printing Office, Washington, D. C., 1942.

Since relief wells must usually depend for their effectiveness upon natural gravity flow, the reduction of losses of head in the structure of the well itself is, as the authors state, a most important consideration. Both the loss in the well casing and the entrance loss through the screen are important. It is recognized that the rate of flow into a relief well would be slower than that into a well pumped for water supply. The losses would therefore be smaller. Nevertheless, experience with wells for water supply has shown that, in the modern gravel-walled type of well, the entrance losses are less than in wells not so constructed. In formations composed of mixed sand and gravels, the gravel wall may be built up by adequately developing the well. If, however, the formation is uniform and fine grained, it may be advantageous to introduce gravel around the screen at the time the well is constructed.

It seems to the writer that relief wells along a levee might present some special maintenance problems. There would be no flow through them over long periods of time. It is conceivable, therefore, that some of the wells in the system might become so clogged by corrosion or by the growth of organisms in the water that they might not be effective when needed in times of flood. Maintenance of the well system would, therefore, become an important part of the maintenance of the levee. Each well should be pumped and tested periodically to assure its continued effectiveness.

Where relief wells are an essential part of a dam, the problem of their maintenance becomes especially important. There is usually a more or less constant head, so that the relief wells will flow continually. It would, therefore, be unnecessary to pump them to determine their continued efficiency if arrangements are made to measure the flow from them periodically. The public and in fact many engineers are inclined to assume that the design of a dam more or less permanently assures its safety. Where the safety of the dam may depend upon the effectiveness of relief wells, the need for competent and continuous engineering supervision of the structure becomes imperative, and the responsibility to the public is greatly increased. Under favorable conditions the life of a properly constructed well might be as much as 40 or 50 years depending largely on the character of the water and the type of materials used in the well. Under less favorable conditions the well might fail within a decade or so. In any event sustained vigilance would be necessary to assure the continued safety of the structure.

In conclusion, the writer believes that this paper deserves the widest publicity and discussion. This relatively new method of increasing the safety of dams and levees should be called to the attention of everyone concerned with the design and maintenance of such structures. In presenting this paper the authors are rendering a distinct service to the profession.

WILLARD J. TURNBULL,[9] M. ASCE.[9a]—Information and design data on drainage wells have been presented in an interesting manner in this paper. Drainage wells have been used on structures of any degree of importance only in comparatively recent times, and this particular engineering feature should

[9] Chf., Embankment, Foundation & Pavement Div., U. S. Waterways Experiment Station, Vicksburg, Miss.

[9a] Received June 27, 1946.

prove of more and greater importance as times goes on with reference to the relief of detrimental substratum pressures beneath and downstream from dams, levees, miscellaneous embankments, and other types of engineering structures. In the not distant future drainage wells probably will be incorporated as a regular feature in the design of all engineering structures built on pervious foundations which are subject to dangerous exit pressures of seepage water.

The question often arises, particularly with reference to structures already built, as to whether drainage wells are needed for the relief of excessive substratum pressures. An effective method in demonstrating the need of a drainage system is by an installation of piezometers. The piezometer installation should be of such magnitude that true gradients of horizontal and vertical pressures can be obtained. Once the true picture of the pressure gradient is known and the actual horizontal and vertical soil profile established, then the engineer will have sufficient data to indicate whether a pressure relief system of some type is needed.

The authors give a very complete discussion of the critical escape pressure gradients from which boils may result, and state that a pressure head loss of from 25% to 50% is not unusual between the upstream and downstream sides. It is desirable to stress that, in some exceptional instances where the top stratum is quite pervious, the pressure head loss may even be as high as from 70% to 80%. If such a condition exists, it is quite obvious that seepage losses under the structure and through the top stratum behind the structure may dissipate dangerous heads with the result that sand boils could not occur. In these particular cases drainage wells would not be feasible since their effectiveness would be relatively small. Drainage wells also might not be feasible in the case of an extremely pervious and thick foundation where the amount of water from the pervious stratum necessary for adequate pressure relief would require a drainage well system beyond all economical limits of construction. In general, on all ordinary pervious foundations with relatively tight top strata, a properly designed drainage system can be entirely effective and feasible, and in many cases may be a necessity to adequately insure the safety of the structure.

In some instances objections may be raised concerning the additional water that is brought to the surface of the ground by a system of drainage wells. It is true, of course, that any system of drainage brings more water to the ground surface and results in additional drainage requirements. In many cases additional drainage requirements can be easily and economically met. However, in some other instances, particularly behind long levees, the drainage problem may be serious from the practical and economical viewpoint and may have to be handled as an independent project. In these latter cases the increased cost of the well system due to drainage will have to be carefully weighed against the reduced hazards attributed to the drainage well system. As a matter of interest, particularly from the psychological angle, the writer believes that if possible all water from drainage wells should be released into a header pipe beneath the ground surface and collection and disposal should be made in a given sump area. An alternate procedure is to have a properly constructed collection ditch which will carry the water to the disposal sump.

The writer would very much like to indicate a difference between the terms "pressure relief" and "seepage relief." In numerous instances these terms have

been used interchangeably although actually there is a vast difference between them. To secure pressure relief—that is, for prevention of boils—an escape pressure gradient of less than 0.6 is certainly adequate in all cases. However, to secure complete seepage relief, a pressure escape gradient of zero is necessary. These facts amply demonstrate why a drainage well system which would give adequate pressure relief may still show considerable free surface seepage. In this connection, the writer wishes to refer to a field experimental installation of pressure relief wells at Sardis Dam at Sardis, Miss. In this installation numerous types of commercial and improvised drainage wells were used. The following is a list of these wells: Improvised—(1) porous concrete (gravel aggregate), bevel joint; (2) porous concrete (slag aggregate), lap joint; (3) cement-asbestos drilled with $\frac{1}{8}$-in. holes on $\frac{1}{2}$-in. centers; (4) square wooden pipe with $\frac{1}{4}$-in. holes on 2-in. centers; (5) perforated concrete pipe with twelve $\frac{1}{4}$-in. holes per foot of pipe; (6) perforated clay pipe with thirty $\frac{3}{8}$-in. holes per foot of pipe; (7) 10-in. steel casing perforated with $\frac{3}{16}$-in. holes on 3-in. by 5-in. centers filled with sand-gravel; (8) 10-in. steel casing perforated as in item (7) with a perforated collection galvanized sheet-iron pipe in the center of the casing surrounded with gravel filter; (9) perforated galvanized sheet-iron tubes with $\frac{3}{16}$-in. holes on $\frac{3}{4}$-in. centers; and commercial—(10) perforated steel pipe with porous cemented gravel filter; and (11) seven commercial well screens with various type slot openings. In all installations of either commercial or improvised wells, a filter was used around the well if the slot opening was of such size that a filter was necessary to prevent the entrance of the foundation sands into the well. In practically all cases the improvised wells have functioned equally as well as the commercially produced products with the noticeable exception of the 10-in. well which was backfilled on the inside with pervious gravel. In this particular case the frictional resistance of the water through the gravel was such that an excessive head loss resulted and in consequence very little water was delivered by the well and very little pressure relief was accomplished.

The U. S. Waterways Experiment Station at Vicksburg, Miss., had previously done considerable work on the design of gravel filters and had arrived at the general criterion that the ratio of the 15% size of the filter material to the 85% size of the foundation material should not be greater than 5. Laboratory tests in a specially designed pressure tank indicated that this ratio might range from 2 to 5, preferably being around 3 to 4. Furthermore, these special tests showed that the ratio of the 85% size of the filter gravel to the diameter of the screen opening should be greater than 1. All filter gravels used in the placement of the Sardis experimental wells were designed on the preceding assumptions and in every case the filters proved to be entirely successful even though in some cases the thickness of the filter around the pipe was not more than 3 in. The necessity for the ratio of the 85% size of foundation sands or filter (as the case may be) to the slot opening (being greater than 1) was demonstrated by the installation of two metal slot wells without filters. In one case this ratio was 1.1 and in the other 0.7. In the former instance the infiltration of sand into the well did not occur, whereas in the latter instance the well became completely clogged with sand.

In the following paragraphs one of two drainage well installations made along Mississippi River levees is described and discussed briefly. At the site of the installation, excessive underseepage accompanied by numerous small-sized to medium-sized sand boils was observed during the 1937 high water. The sand boils were not considered dangerous and only a few were sacked. Heavy underseepage occurred as far as 1,000 ft to the land side of the levee whereas the boil area was confined mainly within 100 ft of the levee toe. A top stratum of relatively impervious silts and silty clays covers most of the area with a thickness ranging from 4 ft to 40 ft. On one low ridge the surface soil consists of a sandy silt which is relatively pervious, and the earliest and heaviest seepage occurred on this ridge. The top stratum is underlain by a fine sand stratum which is quite pervious, and this in turn is underlain by strata of medium to coarse sands and sand gravel, all of which are highly pervious. The average thickness of the top stratum along the line of wells is about 10 ft. The river is about 1,800 ft away. However, large borrow pits exist in front of the levee, some areas of which extend into the fine sand stratum. This fact caused difficulty in arriving at a proper assumption of the over-all average distance of the source of the water from the line of wells. The permeability of the pervious strata was based on laboratory tests obtained on disturbed samples of the material in question. The laboratory tests were used because of the lack of more adequate field information on the in-place permeability of the pervious material.

The installation as made was for test purposes. Consequently, the length of the screen was varied to agree with 10%, 20%, and 30% penetrations of the pervious strata. Every fourth well had a 10% penetration screen at the 20% depth. A minimum clear, inside diameter of $2\frac{1}{2}$ in. was chosen for the well screen and riser pipe. On the basis of the assumptions for permeability and length of flow path and well diameter, a well spacing of 50 ft was used. This spacing did not agree, of course, for the various penetrations but represented an average spacing. The wells used were nonmetallic and consisted of screen, riser, and discharge sections. The screen section was composed of a 4-in. inside diameter porous concrete pipe which served as a filter for a core of perforated clay tile, $2\frac{1}{2}$ in. in inside diameter. The riser section was of solid clay tile, 3 in. in inside diameter, which was connected to a solid clay discharge pipe by a T-section. The riser pipe was carefully sealed through the top stratum by a tamped sand-bentonite mixture. Space does not permit further detailed description of the wells. A surface collection ditch was dug near and parallel with the line of the wells for disposal of the water.

A system of piezometers was installed. The system consisted of a single line of piezometers in the line of wells with several lines running transverse both to the land side and river side of the levee.

During the spring flood season of 1943 a relatively low head of about 7.5 ft was experienced against the levee at the location of the wells. A very brief summary of the test findings follows:

a. The piezometers in the line of wells with all wells closed indicated that about 70% of the river-side head was dissipated by underseepage alone and that with the wells open an additional decrease in head of 10% was obtained. These

data indicate that the top stratum at the site of the test installation was too pervious to demonstrate the effectiveness of a well system adequately.

b. Test data indicated that the well discharges obtained were about twice those expected for the given head, thus indicating either that the assumed coefficient of permeability was not great enough or that the effective distance to the source of the water chosen was too great. These facts indicate the necessity of designing a well system with a fairly large factor of safety.

c. Test data indicated that the friction losses obtained in the well, from $2\frac{1}{2}$ in. to 3 in. in inside diameter, were too great for the quantity of water produced and that larger clear diameters were necessary.

d. The nonmetallic type of well as installed is highly efficient in the production of water but needs further thought and study to insure that the wells do not develop faults caused by the many joints which are not positively connected.

e. Test data indicated that the 10% penetration well was relatively ineffective but that, if the wall were placed at the 20% penetration depth, it would produce about as much water as the full 20% penetration well. The 30% penetration well is appreciably more effective than the others, showing that a minimum penetration of at least 25% may be desirable in most cases.

f. Test data were not conclusive because of the relatively low head; higher heads are needed for final and conclusive data.

g. Test data indicated that the lack of information on actual field in-place permeabilities is serious and every effort should be made to determine this value. (An excellent method of determining the field permeability is by a large-scale field pumping test and alternate field procedures are the dye or electric methods; however, in general, determination of permeabilities by pumping water into a stratum is not recommended.)

h. The outlet of drainage wells should be as low as possible.

The writer wishes to emphasize a few features of the design and installation of drainage wells. Extreme care should be taken to secure a tight backfill around the well through the top stratum material. This measure is obviously necessary because the higher pressure of the deeper and usually more pervious strata will be brought directly to the surface through the medium of the well and should the well in any manner become plugged, there may be material danger of piping around the well. Thus, every precaution should be taken so that a drainage well is not plugged in any way while in operation. Another feature of extreme importance in the design of drainage wells is adequate insurance that the diameter of the drainage well is sufficiently great that friction and velocity head losses are maintained at a minimum. It is quite possible that the difference between an effective drainage system and one that is ineffective might be a difference of 1 in. in diameter of the wells. To insure the proper functioning of a drainage well system, it is very essential to remember that a properly designed system may be rendered ineffective by a poor installation. The authors have placed needed emphasis on the importance of obtaining the best information possible concerning (1) proper evaluation of the over-all coefficient of permeability of the pervious strata, and (2) the effective distance between the source of the water and the line of wells.

GLENNON GILBOY,[10] Assoc. M. ASCE.[10a]—A neat summary of the present state of development, both theoretical and practical, of well relief systems is presented in this paper. Although much remains to be learned, such systems have already proved to be of great practical value, and probably they will have an increasingly wider range of application.

Compared with rock toes, gravel drains, and other surface relief measures, the drainage well has outstandingly useful characteristics. Not only does it serve to penetrate a thick impervious blanket at much less expense than trenching, but it also serves, in the pervious stratum itself, to intercept a whole succession of layers, thus providing relief where needed and compensating for the inferior drainage characteristics of stratified deposits in the vertical direction.

The importance of these effects can readily be visualized by considering, as an extreme case, a pervious deposit composed of thin layers of sand separated by layers of tar paper. It is obvious that a surface drain would afford practically no relief, whereas a well, punching through the layers of tar paper, would be very effective. The formation of a boil would correspond to a successive tearing of the tar paper layers, beginning at some weak point, as a result of the horizontally transmitted pressures. The surface drain would not prevent this, whereas a properly designed well system would reduce the pressures so that the progressive rupture could never get a start.

Natural deposits do not normally reach this extreme, but the effect is present to some degree in practically all sedimentary materials. Therefore, the applicability of theoretical analyses and model tests based on homogeneous and isotropic media is limited. As guides such analyses and tests are useful, but it is too much to expect that their results will be reflected quantitatively in the prototype. Thus, although wells are peculiarly adapted to drainage of stratified deposits, the existence of stratification throws a considerable element of uncertainty into the design. It is proper to recognize this condition at the outset and to be prepared to modify the installation if the first trial proves inadequate. Fortunately, as the authors state, well systems are naturally flexible, so that the cut-and-try method is not as much of a drawback as it is in other types of drainage installations.

The authors speak of controlled versus uncontrolled drainage. This concept is of prime importance. In certain cases it is desirable, and often necessary, to make a dam as nearly watertight as possible; but there are many instances in which a high degree of watertightness not only is expensive to obtain, but also is not in the least necessary for the functioning of the structure. In structures intended merely to retard, rather than completely stop, the passage of water, a design which contemplates substantial seepage losses can be made entirely adequate by proper drainage control.

Control of seepage through the body of the dam itself is usually provided by a rock toe, a horizontal drainage blanket, or other suitable method. The factors governing the flow pattern and the quantity of seepage are reasonably well understood, so that the drainage system can be planned intelligently. Good modern practice in this respect, however, is by no means universal.

[10] Cons. Engr., South Lincoln, Mass.

[10a] Received August 30, 1946.

Although the subject has already been well reviewed, within the past five years a flood control dam, with normally dry reservoir, has been constructed with a 1-on-3 upstream slope and a 1-on-6 downstream slope—the avowed purpose of the latter being to keep the line of saturation covered up. The fact that this queer concept is still extant shows the need for continued emphasis on the subject of sound drainage control.

Control of foundation seepage is every bit as important, and is frequently much more complicated. Sometimes it is possible to combine the control systems, draining the foundation into the same element which drains the dam. Sinking an underdrain through an impervious natural blanket and connecting it to the rock toe of the dam is a good example of this practice. On the other hand, there are many cases in which this procedure is impractical, or, if adopted, would prove ineffective. In this region, the drainage well comes into its own. The well should be considered, therefore, not merely as an adjunct or a corrective measure, but as an element in the design, coordinated with the other elements of the drainage system. The Arkabutla installation is an important advance in this field, and there is every reason to believe that future designs will benefit from a similar approach.

The factual material presented by the authors on existing installations is interesting and instructive. The writer has no comment to make, except to suggest that, for those cases which have not as yet been subjected to extreme operating conditions, the authors present a follow-up when those conditions arise, so that the profession may be informed of the final adequacy of the installations and of the additional corrective measures, if any, which the extreme conditions may require.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ENGINEERING EDUCATION

PROGRESS REPORTS OF THE SOCIETY'S PROFESSIONAL COMMITTEE ON ENGINEERING EDUCATION, FOR 1944 AND 1945

Discussion

BY HARRY RUBEY

HARRY RUBEY,[7] M. ASCE.[7a]—The reports are well conceived and well executed. They will be regarded highly by educators who are faced continually with the responsibility of "maximizing" the preparation of young engineers.

In interpreting this valuable summary of opinion, it will be useful to group certain subjects. For example, if highways, railways, and airways are combined under the single subject of "transportation," this broader classification would justify a much higher rating than has been allotted to each separately. Courses in "transportation," as given by a particular institution, would doubtless emphasize one or another of the foregoing separate subjects to meet current employment opportunities—at the same time covering all three subjects in at least an introductory manner. The Army Specialized Training Program included a course of this nature entitled "Transportation 403."

A more significant grouping might be made of certain items in Tables 3(*b*), 3(*c*), and 3(*d*) which include a large number of subjects and to which others might have been added. It is obvious that separate courses cannot be given for all the nontechnical subjects that have relatively high ratings, although most of these are important in varying degrees to any particular graduate. Some of these subjects will be touched on in the usual technical courses, but there is need for a comprehensive senior course that might be termed a "Survey of Management for Engineers," in which the student could be introduced to most of the following subjects, and to other phases of management:

From Table 3(*b*), Basic Sciences—statistical analysis;

From Table 3(*c*), Other Subjects—professional relations, personnel and labor relations, engineering economy, public administration, ethics, finance, industrial management, industrial history, and accounting; and

From Table 3(*d*), Civil Engineering Subjects—valuation and appraisal.

NOTE.—These reports were published in March, 1946, *Proceedings*. Discussion on this report has appeared in *Proceedings*, as follows: June, 1946, by Frederic Bass, and John B. Wilbur; and September, 1946, by Lowell O. Stewart, and Roy M. Green.

[7] Chairman, Dept. of Civ. Eng., Univ. of Missouri, Columbia, Mo.

[7a] Received July 31, 1946.

Sufficient introductory information and appraisal of many of the subjects listed in the preceding paragraph, and others, can be presented in a single general course. If conducted in conferences rather than by lectures, such a course will assist in overcoming the major deficiencies found in graduates. So taught it will widen horizons; and it will increase effectiveness in conference speaking, in formulating problems of a broad nature, in considering all cost factors, and in participating in public affairs. Perhaps such a course could be considered as contributing toward the humanistic-social stem, since business and management situations are used to develop an appreciation of the over-all manner in which people organize and conduct their activities.

Revelations of the deficiencies in engineering graduates are an important feature of the reports, and they define the area in which constructive steps should be taken. It is apparent that the deficiencies mentioned result in part from the narrow and specialized viewpoint which is difficult to avoid in most technical courses. In so far as engineering education can alleviate these shortcomings, the improvement must be accomplished through courses that the students like. Innate characteristics of the engineer cannot be changed materially.

Engineering students are as much interested in management and allied subjects as they are disinterested in the courses given by the arts college. Practicing engineers have expressed a similar evaluation, under the heading, "Opinions Expressed in Questionnaire," in the 1945 report and in Fig. 3 and Table 3 of the "Appendix." It has long been known that the majority of civilian engineers engage in work that is primarily administrative. Commissioned engineer officers noted during the war that their duties were largely administrative. Thus, overwhelming corroborative evidence and natural motivation both indicate the need for a general survey course based on management as perhaps the best single academic remedy for overcoming the deficiencies found in engineering graduates.

If the observed handicaps are to be corrected, educators must take advantage of the interest of the students and of the opinions of the older engineers as summarized in the reports. These handicaps have long been known or suspected but little has been done about them. Obviously, past educational procedures have not been effective.

It was probably not entirely clear to those hurriedly answering the questionnaire whether the ratings of subjects and deficiencies were meant to apply to new graduates from the employer's viewpoint, or to the preparation of the individual himself for his later years. Doubtless the former interpretation predominated. Had the latter view been taken, the ratings on management subjects and abilities would have been higher, especially if they were grouped together, since the duties of the older engineers are mainly administrative.

It must not be overlooked that most graduates will receive only the bachelor's degree and are not adequately educated for a modern, highly technical career in which graduate preparation is increasingly necessary. Most of these four-year men will therefore be limited to high-grade technician positions, or they must turn toward management and semitechnical activities where there is less of a "ceiling" to their advancement and where they are in great demand.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TOPOGRAPHIC SURVEYS

PROGRESS REPORT OF THE COMMITTEE OF THE SURVEYING AND MAPPING DIVISION ON TOPOGRAPHIC SURVEYS

Discussion

BY ROGER E. AMIDON, DANIEL KENNEDY, AND RALPH P. BLACK

ROGER E. AMIDON,[1] Assoc. M. ASCE.[1a]—Under the heading, "Purpose and Scope," the committee expresses the hope

> "* * * that the report will tend to raise the standards of topographic surveying and will help engineers to appreciate the fact that topographic mapping is a type of service that requires special training and that offers opportunity for the display of skill and imagination in the improvement of the quality of the service."

The report undoubtedly will assist in accomplishing this result. Certainly there is enormous room for raising the standard of topographic maps. An improvement can be effected by teaching the users as well as the makers of maps the difference between good and bad maps, the economic advantage of good maps, and how they can be made economically. The report is a beginning in this direction.

Civil engineering is such a broad subject that no man can possibly be expected to be an expert in many phases. Topographic engineering is a highly specialized field and should be considered as such. Surveying, including topographic mapping, is one of the oldest phases of civil engineering and is perhaps taken for granted by too many engineers. Engineers in charge of projects should be continuously on the alert for inferior surveying and mapping work on which they are to base their designs and estimates. The tendency to feel that any man who has had basic instruction in surveying is a qualified topographer and needs no supervision, training, or technical assistance is widespread among engineers and others. If the project engineer understands this fact and guards against it by employing competent supervisory mapping per-

NOTE.—This report was published in April, 1946, *Proceedings*.

[1] Associate Engr., U. S. Forest Service, Altadena, Calif.

[1a] Received June 25, 1946.

sonnel with leadership and administrative ability, the efficiency of the project will be expedited and the quality of his final results will be materially enhanced.

Good general and detailed topographic maps can be made by comparatively inexperienced personnel under proper supervision. The average young surveyor can be trained on the job in a comparatively short time to do work of a good quality with reasonable speed. A highly skilled topographer can supervise and train from four to eight field plane-table parties with inexperienced party chiefs, and within a period of from two to three months have them turning out a satisfactory quality and quantity of work. Training should include field demonstration and instruction. Supervision should consist of the detailed planning of each man's work as well as the solution of all new and unusual problems arising. Many short cuts are developed by all topographers as their experience is increased. Many of these can be explained, demonstrated, and discussed with the trainee. Good professional morale is essential to the success of any individual in any undertaking and certainly holds true in topographic mapping. An enthusiasm for the work must be developed in the trainee to bring out his best efforts. This result can be accomplished by presenting, in action and word, the importance of a good map and the high degree of skill attainable. Actually, topographic mapping embodies art as well as engineering. The satisfaction derived from the completion of a technically topographical and artistically correct topographic sheet is similar to that obtained by the artists from a completed painting.

In comparing the various methods of mapping, much more can be mentioned regarding their advantages and disadvantages. Textbooks are glaringly lacking in presenting definite information on this subject. In general, engineers are more familiar with transit work than any other type of surveying, and therefore many assume that it is adaptable to every possible condition of mapping. It is probably fair to state that more general and detailed maps are made by the transit method than by any other procedure. Practically the only advantages that this method offers over others are that more engineers understand the mechanical operation of the transit and it is the most common and only basic instrument needed in the preparation of this type of map. The first of these advantages is of negligible value on a mapping project of any magnitude, in view of the fact that capable plane-table men can be trained in a short time. The second is of insignificance when the cost of a plane-table outfit is considered. The entire cost of an alidade, tripod, and board will be offset by a reasonably competent operator in a period of from one to three months.

Many of the transit maps of general and detail scale are still prepared by transit cross-section method. Qualified and competent engineers in other fields fail to take the time to diagnose the inherent failure of this type of mapping. It is common practice to take cross sections at 25-ft, 50-ft, and even 100-ft intervals for the preparation of maps at large scales of from 1 in. equals 50 ft to 1 in. equals 100 ft and contour intervals of from 2 ft to 10 ft. The glaring deficiency of this method even as compared to random shot transit topography is evident when it is realized that rod positions at a scale of 100 ft to the inch are, on most terrain, spaced at a maximum of 25-ft intervals with additional

shots taken to show detail that cannot otherwise be properly presented. Some engineers believe that reservoir quantities, for instance, are not of sufficient accuracy when determined from contour planimetry and that volumes determined by cross sectioning are superior. The reason for this erroneous conception is obvious when one considers that ordinarily repetitive cross sections are taken at previously sectioned and marked stations and consequently show less discrepancy from year to year than do cross sections taken at random and compared to cross sections plotted from the topographic base map. It is the writer's contention that the over-all most accurate and satisfactory method of determining the capacity of reservoirs is from a good base map at correct scale and contour interval. Subsequent reservoir capacities will be correctly determined by measurement from a new contour map of the areas where change has occurred.

In general, the accuracy of a map is directly proportional to the number of correctly placed shots over the whole area. The cross-sectional method usually ignores minor changes of shape and planimetry. The additional work involved and the increased chance for error usually drastically handicap transit topography and favor plane-table methods. The office engineer must be an extremely capable individual if he is to interpret transit field notes correctly. The field engineer must be an experienced, capable, and imaginative person if he is to take only the shots needed to correctly present the topography of the area. A plane-table instrumentman soon learns that unnecessary shots are a waste of time and effort, and a shortage of vertical and horizontal control positions results in the impossibility of correctly presenting the terrain.

Without question, the combination of aerial photographs and plane-table methods is most advantageous where photographs are available or obtainable and the size of the project economically justifies their procurement and use. This method should definitely be considered on projects of any appreciable magnitude.

Costs are usually disconcerting to the topographer with the artist's desire for perfection—the tendency may be to attempt accuracy and detail beyond the need of the specific map—particularly detail scale maps. The specifications for accuracy suggested in Section C of the Appendix visualize the presentation of all detail within drafting limits. This is, undoubtedly, academically desirable; but, as a rule, in practice it is subject to radical divergence. The individual purpose for which the map is made determines the accuracy necessary. The design engineer is not accustomed to reading detail to the fineness with which it can be easily drafted by the competent topographer. A scale of 1 in. to 10 ft or 20 ft with 1-ft contours may be desired by a designer so that he can plot reasonably accurately the outline and elevation of structures. The correct position of topography and planimetry to a drafting accuracy of 1 in. to 100 ft is usually sufficient for his purpose although, for convenience, the larger scale is desired.

Engineers in general do not fully understand the results that can be expected from enlarging maps. The United States Geological Survey quadrangle sheets are frequently criticized because, after enlargement from four to eight times, they include areas that do not agree with surveys compiled on the new large

scale. It is more satisfactory to use a field scale on general and detailed scale maps of sufficient size to take care of any anticipated scale needs rather than to enlarge the map after completion.

Many maps are made without regard to previously established datum. This is, of course, undesirable but it is difficult to present to the individual the economic advantage of doing extra work for the purpose of showing correct horizontal and vertical positions on an isolated area which is prepared and used for a specific immediate purpose. In many cases, of course, the small amount of extra effort necessary to tie in a map to true horizontal and vertical control positions is repaid many times in a short time. Certainly all local, state, and federal agencies should be required to tie in maps prepared by them correctly. The long-range advantage of this can scarcely be questioned although at present it is not universally enjoyed. The time and effort lost in remapping and endeavoring to correctly tie in previously mapped areas would, over all, more than pay for the original permanent establishment of markers. An appeal to the professional integrity of engineers should be beneficial.

The practice of many organizations is to provide the topographer with the materials and instruments he needs, show him the area he is to map, and turn him loose. Instances are known in which the topographer was not furnished with the scale, contour interval, or use to which the map was to be put prior to his initiation of the work. General specifications should be prepared in writing and furnished to the engineer in charge of surveys for his guidance in supervising the work. He, in turn, should prepare detailed specifications and instructions for the information and guidance of the party chiefs. A complete understanding by the party chief of the quality of work expected and the method of inspection and checks that are to be used will keep him alert and conscientious as well as give him an understanding of the quality and quantity of work demanded.

Inefficiency in field mapping is the rule rather than the exception. Engineers unfamiliar with topographic mapping will send mapping parties into the field with more men than are desirable for efficient operation. The party chief may realize this fact, if he is an experienced mapper, but hesitates to make a point of it as a reduction in personnel may increase his work and also he hesitates to tell his superior how the job should be administered. Plane-table parties can seldom utilize more than three men. The party chief (who is instrumentman), a recorder, and one rodman. The recorder can be dispensed with in small-scale mapping. Two rodmen are seldom, if ever, efficient on plane-table work. On detail scale maps, rod positions are spaced at such close intervals that one rodman will more than keep the instrumentman busy. On small-scale maps, one rodman will keep the instrumentman occupied except where the terrain and vegetation are extremely difficult. The experienced topographer is able to circumvent many small areas of this nature. Efficient operation requires continual and detailed planning on the part of the mapper and the rodman to get necessary shots in the shortest time and with the least effort. Many brush lines are cut which are not necessary and could be omitted by other methods of approach. Additional instrument positions can usually be planned to cover the area. Side shots beyond the range of vision of the topo-

grapher are dangerous to accuracy unless the terrain is regular or unless the rodman is sufficiently experienced and familiar with the topographer's methods to describe the features accurately. It is usually more accurate and time saving to utilize additional instrument setups than to cut brush. Much expense is caused in most topographic surveys by lack of experienced rodmen. The difference between a good experienced rodman and an inexperienced one can easily result in a variation of from 10% to 20% in the quantity of production as well as in a reduction in the accuracy of the results. If project managers realized this, additional pay for good rodmen would be more readily forthcoming.

In transit topography, two rodmen, a recorder, and an instrumentman (who is the party chief) are sufficient on most surveys. Many engineers consider the plane table an awkward instrument to maneuver in difficult terrain. There is little foundation for this contention if the correct board is used. A 15-in. by 15-in. board can be maneuvered in the most rugged vegetation and terrain without undue difficulty by an experienced man. Boards, 9 in. by 9 in., have been used on small-scale mapping. Celluloid sheets are very effective in climates where rain causes difficulty in the use of paper.

A summary of the pertinent facts in this discussion is as follows:

1. A need exists for the education of users of topographic maps.
2. Good general and detailed maps can be made by comparatively inexperienced personnel with proper administration and supervision.
3. Plane-table methods on any scale are usually more economical and result in a higher degree of accuracy, if properly made, than do other ground methods.
4. Specifications and planning throughout all parts of the project are essential to efficient and quality mapping.
5. Personnel requirements for efficient mapping should be high.

The writer agrees, in detail, with the report and desires only to augment it.

Daniel Kennedy,[2] Assoc. M. ASCE.[2a]—Topographic surveys, as treated in this report, cover the subject of large-scale project mapping very well. The report provides the private engineering survey organizations with a clear idea of specification requirements, and serves as a firm guide to the construction engineer and architect who use this type of survey. The specifications are confined to construction projects, however, and, as such, do not apply to topographic mapping in the general use of such terms in national mapping. As an example, 1:12,000 would be classed as a small scale within the scope of this report, whereas in general mapping 1:12,000 is considered a large scale. The classification of such maps, as defined, is satisfactory. However, a map of 1:4,800 can well be used for a detailed construction survey as was done during the construction of Fort Leonard Wood, Missouri, in 1941.

Selection of Map Scales.—In projects of this type cost is a large factor. As such, a closer relation should be observed between physical factors and scale rather than the comparison of one scale against another.

[2] Chf., Operations and Planning Staff, Army Map Service, Washington, D. C.

[2a] Received August 21, 1946.

sonnel with lead
will be expedited

Good general
inexperienced p
veyor can be tr
good quality wit
vise and train
party chiefs, and
out a satisfactor
demonstration
planning of ea
problems arisin
experience is i
discussed with
of any individ
mapping. An
bring out his
action and
attainable.
ing. The sa
cal and artis
artists from

In compa
tioned regar
ingly lacking
engineers are
and therefore
mapping. It
made by the
only advanta
understand th
and only basic
first of these
magnitude, in
short time. Th
outfit is consid
offset by a reas
months.

Many of the
transit cross-sectio
fields fail to take th
ping. It is common
intervals for the pr
to 1 in. equals 100 ft
deficiency of this meth
is evident when it is
are, on most terrain,

grapher are dangerous to accuracy unless the terrain is regular or unless the rodman is sufficiently experienced and familiar with the topographer's methods to describe the features accurately. It is usually more accurate and time saving to utilize additional instrument setups than to cut brush. Much expense is caused in most topographic surveys by lack of experienced rodmen. The difference between a good experienced rodman and an inexperienced one can easily result in a variation of from 10% to 20% in the quantity of production as well as in a reduction in the accuracy of the results. If project managers realized this, additional pay for good rodmen would be more readily forthcoming.

In transit topography, two rodmen, a recorder, and an instrumentman (who is the party chief) are sufficient on most surveys. Many engineers consider the plane table an awkward instrument to maneuver in difficult terrain. There is little foundation for this contention if the correct board is used. A 15-in. by 15-in. board can be maneuvered in the most rugged vegetation and terrain without undue difficulty by an experienced man. Boards, 9 in. by 9 in., have been used on small-scale mapping. Celluloid sheets are very effective in climates where rain causes difficulty in the use of paper.

A summary of the pertinent facts in this discussion is as follows:

1. A need exists for the education of users of topographic maps.
2. Good general and detailed maps can be made by comparatively inexperienced personnel with proper administration and supervision.
3. Plane-table methods on any scale are usually more economical and result in a higher degree of accuracy, if properly made, than do other ground methods.
4. Specifications and planning throughout all parts of the project are essential to efficient and quality mapping.
5. Personnel requirements for efficient mapping should be high.

The writer agrees, in detail, with the report and desires only to augment it.

DANIEL KENNEDY,[2] Assoc. M. ASCE.[2a]—Topographic surveys, as treated in this report, cover the subject of large-scale project mapping very well. The report provides the private engineering survey organizations with a clear idea of specification requirements, and serves as a firm guide to the construction engineer and architect who use this type of survey. The specifications are confined to construction projects, however, and, as such, do not apply to topographic mapping in the general use of such terms in national mapping. As an example, 1:12,000 would be classed as a small scale within the scope of this report, whereas in general mapping 1:12,000 is considered a large scale. The classification of such maps, as defined, is satisfactory. However, a map of 1:4,800 can well be used for a detailed construction survey as was done during the construction of Fort Leonard Wood, Missouri, in 1941.

Selection of Map Scales.—In projects of this type cost is a large factor. As such, a closer relation should be observed between physical factors and scale rather than the comparison of one scale against another.

[2] Chf., Operations and Planning Staff, Army Map Service, Washington, D. C.

[2a] Received August 21, 1946.

Classification of Maps According to Accuracy.—Map accuracy should be a function of the scale—that is, the limiting plottable distance should be accuracy for that particular scale. It then follows that at a scale of 1:12,000 the accuracy would be approximate to the nearest 10 ft in position. Since 0.02 in. would be the limiting plottable error, at a scale of 1:240, this would equal 4.8 in. Using standard orders of survey accuracies, then, it would be a simpler matter to decide which order would suffice for a particular scale.

At times auxiliary contours are necessary to depict the ground. However, unless used with discretion, they are likely to confuse the map readers. It is better to select an interval and "stay with it."

In writing specifications for mapping work, the writer should always bear in mind the local field problems as well as the use to which the map will be put. The topographer should have some latitude in interpreting the requirements and should not be restricted to certain specified means of action. Proper supervision and inspection will accomplish much toward speed of completion and a homogeneous interpretation of detail.

Method of Development of Topographic Maps.—As stated, several methods are available. The writer's preference would be the plane table, supplemented by air photos if possible. It is practically impossible to photograph areas at low enough altitudes to obtain the required contour interval for a map scale of 1:4,800 or larger. However, a topographer can use pictures flown at a higher altitude for planimetric detail and place the contours on the sheet by field methods. As also stated in the report, the topographer can work on the photos; but usually the relief on a photo will cause enough displacement to cause errors above the plotting scale.

Transit stadia method on certain areas will be satisfactory, but its use on other areas will result in an entirely wrong interpretation of the ground. Any survey that is tied into national or state systems should be monumented. The smaller ones that have an assumed datum can just as well be tied with metal hubs or stakes.

Map Reproduction.—The subject of map reproduction is covered sufficiently well in the report. Usually the blueprints are made because of an absence of camera facilities. Whenever possible, photographic reproduction should be made with glass negatives because even the best topographic base film is dimensionally unstable.

Wherever possible, the map should be correlated with contiguous controls. The use of state coordinates will add to the accuracy of the work by giving a check to the survey by tying in another part of the state system. Often, enough points will be available to dispense with the requirements for an astronomic observation.

The control specifications are well written. There appears to be one question about the vertical closure as given in paragraph *A–10(g)* of the "Appendix." The closure at $0.03\sqrt{\text{distance}}$ is of second-order accuracy, whereas most leveling for 1-ft or 2-ft contours would only require $0.05\sqrt{\text{distance}}$, or third-order accuracy.

Ralph P. Black,[3] M. ASCE [3a]—In its analyses, and in the information given, this report covers the assigned subject very effectively. The instructions for procedure, degree of accuracy, and methods, are well thought out and are most helpful to the surveying profession. The specifications for making topographic maps for the first three clases of maps, and methods of making them, are well stated. In an objective analysis of the report the writer proposes to discuss the classification of detailed precision topographic surveys, and the details of the actual field work by the survey party. On work of this type the writer has used the following procedure and methods:

First, the engineer determines the main use that the topographic map is to serve. The client may say, for example, that the architect requires a 1-ft vertical interval and the location of all data needed for a building project. The second step is to decide the scale, and the degree of accuracy, required for the job.

The field party for small areas should comprise the engineer, instrumentman, two rodmen, and two chainmen. A closed traverse is run, and property lines are established. Tied in with this traverse are secondary base lines laid out on ridges, from which hand-level lines are run at right angles, normal to contours. The party then runs a closed line of levels and establishes the elevation of the ground at all points where reference bench marks may be needed.

The third step is to locate, on the ground, all contour lines called for on survey. The transit is used to keep the hand-level party at right angles to each base line. A telescopic hand level should be used. The rodman then moves to a new point where the difference in ground gives the direct location of the contour elevation of the next vertical interval; that is: The computed rod reading for this interval is read by hand level. The distance is read by a chainman using the base line as station 0 + 00.

The system of plotting ground points by means of right angles measured from given base lines affords a quick means of mapping the correct location of contours. All errors resulting from interpolation are thus eliminated.

For map scales of 1 in. equals 100 ft, the writer has found that a sketch board used in connection with a transit-stadia survey is an efficient method of locating topographic detail.

The closed horizontal and vertical control traverses are run as stated in steps one and two and all transit points to be used for sketching are tied in to these controls. A field party then consists of a sketch maker or map maker and a transit-stadia party made up of an instrumentman, two rodmen, and one computer. The stadia "shots" are observed from convenient stations to locate all necessary ground points, which, in turn, are plotted by protractor. Contour lines are located by interpolation.

The foregoing procedure is better and more accurate than the usual plane-table method.

---

[3] Cons. Engr., Associate Prof., Civ. Eng. Dept, Georgia School of Technology, Atlanta, Ga.
[3a] Received August 28, 1946.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MOMENT-STIFFNESS RELATIONS IN CONTINUOUS FRAMES WITH PRISMATIC MEMBERS

Discussion

BY I. OESTERBLOM

I. OESTERBLOM,[2] M. ASCE.[2a]—The idea expressed in the title of this paper is excellent; and the author promises a development that is much needed for a quick adjustment of moment schedules of a framework to which changes are made—particularly, in industrial construction where loads so often must be shifted and sections changed. It remains to be demonstrated that the paper fulfils that promise. The third paragraph of the "Synopsis" and the examples that follow it suggest that the author may have a broader scope in mind: A universal method as a convenient substitute for the Hardy Cross relaxation method.

An important truth appears in the statement (second paragraph of the "Synopsis") that

> "Since the moment at a given section in a continuous structure is a function of the stiffnesses of the various members, the differential of the moment (or moment differential) is a function of the differential changes in the stiffnesses."

but this statement will not help very much unless the functional relation between the moment and all the sections (or the functional relations between one section and all the moments) can be given a very simple and true expression. In view of this fact, both the culminating Eq. 7 and the first sentence of paragraph three in the "Synopsis" seem incongruous. The complex appearance of Eq. 7 is most repellent; and to establish the moment-stiffness relation independently would mean the application of a few cycles of relaxation to the entire frame for each unit couple or unit force. How utterly impossible this task is—in a commercial sense—is not apparent from the paper, which demonstrates only primitive frames; and it is seldom that an engineer is given such simple tasks. This is true for all the examples except the bents in the "Appendix."

NOTE.—This paper by George H. Dell was published in May, 1946, *Proceedings*.

[2] Engr., Carbide & Carbon Chemicals Corp., South Charleston, W. Va.

[2a] Received July 17, 1946.

Not only is Eq. 7 unattractive, but the question may also be asked: Is it correct? Professor Dell declares cautiously (last paragraph of the "Synopsis") that the "* * * differential quantities are only approximate representations of the true changes." If it is correct, is it useful? Possibly it is useful for design but not for the correction of moments. The basic formula, Eq. 1, was differentiated to the form

$$dM_{AB} = \Psi_{AB}\, dK_{AB} + K_{AB}\, d\Psi_{AB} \quad \text{(24)}$$

—leading to Eq. 2. This differentiation is permissible only if the curves of the slowly changing values of K_{AB} and Ψ_{AB} can be given a functional expression in relation to one another or to a fourth variable. What Professor Dell has in mind, seemingly, is not this condition but a rather violent quantum variation of K_{AB}, and thus also of Ψ_{AB}; such a relation does not permit differentiation. It could be done, of course, if the functional relation had been expressed. The sequence of developments that follows, in the paper, is correspondingly suspect, and the superposition that leads to Eq. 7 is entirely out of order. Therefore. Eq. 7 represents mere approximation at best; and it may be seriously in error.

Assuming, however, that it is reasonably true; how useful is it? Can a moment at any point be found independently by taking advantage of the stiffness elements of the entire frame? Can moment corrections be applied at all points by the section adjustment at a few points? The profession will be grateful for Professor Dell's answer to this question, because it will have considerable economic significance.

An interesting challenge to test the paper would be its application to the example in Fig. 1, expanded to a three-span, seven-story frame with fixed foundations. Can the author write the moment-stiffness relations for all points of such a frame? This is the kind of frame an engineer encounters intermittently and quite often. Applying the Dell method, an analyst would have his choice (1) of computing all the nodal point moments (if this is what he needs in the design) or (2) of changing the size of three or more of the members in the frame (this is what often happens) and computing the changes to all the nodal point moments due to the change of size (if he prefers).

In the solution of any similar problem there is a fundamental difficulty. The moment at any point is not only an aggregate of superposed moment elements, but also—in a philosophical sense—a divergent series and a very unusual one, which is difficult to set up and quite impossible to integrate. It is fanlike and interwoven—fanlike, because it is a complex assortment of many subordinate series; and it is interwoven, because these subordinate series are intermittently associated with, and assisting, one another. At the same time each one converges separately toward zero.

Naturally, this concept is to be taken as a philosophical ideal only. It is doubtful if any one will ever be able to give definite mathematical expression to the accumulation of moments along their devious paths of approach; but the idea should be helpful nonetheless, both as a guide as to what still may be done and as a warning as to what should not be attempted. The history of kinetics is ample proof how true this is. The great pioneers—Clapeyron, Maxwell, Greene, Otto Mohr, Castigliano, Williot, Manderla, Fidler, Müller-Breslau,

Ostenfeld—all made important contributions to theory and method; but they were limited in their applications to continuous beams or arches, and to simple frames.

A number of years ago, the writer had occasion to assign a similar problem to a colleague, distinguished for his grasp of all the advanced methods of structural analysis. The structure was a three-span, three-story, reinforced-concrete building with different spans and story heights and with concrete elements fully designed. There were different uniformly distributed floor loadings on each span, one load concentration on each of the three floors, and an applied wind load—in all possible combinations. The problem was to determine all the maximum dead-load and live-load moments at the nodal points and in the girders.

This colleague had the choice of any one of the classical methods available at that time (and he knew them all); but after three months of study he abandoned the problem as being beyond the reach of human effort.

It was only when the revolutionary method of relaxation was introduced by Hardy Cross,[3] M. ASCE, and R. V. Southwell[4] that one succeeded—with fair economy of labor and complete accuracy—in computing the moments in a really complicated modern frame. The reason why this method succeeded lies also in the underlying philosophy of the problem. By its progressive and alternate freezing and releasing of joints, the relaxation method superimposes a method that is synchronized to the difficulties of the problem; it is thus that the innumerable redundants, hidden in as many equations and set up in a hopeless matrix of determinants, are eliminated. There is only one of the earlier methods to which this does not apply: The graphical method by Fidler, later developed by Müller-Breslau for continuous beams and arches and by Strassner for continuous frames. This method also has given designers a device for mastering the difficulties, and it should be used more extensively.

The critical parts of this discussion are not directed to Professor Dell only; but to the many others who have tried to eclipse the Hardy Cross method by offering a better one. No doubt details of procedure can be improved upon; but until a definitely superior method is discovered, which fits the difficulties of the problem better than the relaxation method, it is mere waste of time and paper to try the impossible. Details of procedure can be improved: Many already have been offered and accepted by the profession. Those who aspire to be pioneers might be more successfully constructive if they worked along these lines and also with the comparatively unknown graphical solutions.

If Professor Dell merely intended to present a rough and approximate solution for simple frames the answer is that a one-cycle relaxation will do equally well; it will be just as rapid—if not more so. Two cycles will be required for heavy point loads only, especially if they are near the supports. Even the famous old equation offered by Clapeyron in 1857 would do as well; and in most such cases it would be better than Eq. 7 proposed by Professor Dell.

---

[3] *Transactions*, Vol. 96, 1932, p. 1.

[4] "An Introduction to the Theory of Elasticity," by R. V. Southwell, Oxford Univ. Press, 1936, p. 91.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CRITICAL STRESSES IN A CIRCULAR RING

Discussion

By Robert H. Philippe and Frank M. Mellinger

Robert R. Philippe[15] and Frank M. Mellinger,[16] Assoc. Members, ASCE.[16a]—The development, by Messrs. Ripperger and Davids, of L. N. G. Filon's formal solution to evaluate the critical stresses in circular rings is an excellent contribution, particularly in the testing of brittle materials. The original application of the "ring test" is attributed to Max M. Frocht of the Carnegie Institute of Technology, Pittsburgh, Pa., who had used this type of test to measure the tensile strength of bakelite. Inasmuch as the shape of specimen required is easily cut and drilled from cylindrical cores, the test was adapted to the testing of concrete and foundation rock in connection with the design of concrete dams. More recently the test has been applied with promising results to the evaluation and correlation of aggregate quarry cores. However, this discussion will be limited to the application of the ring test in determining the physical properties of the more massive types of foundation rocks.

If the tensile strength of a brittle material is determined by a ring test and its compressive strength is measured by an unconfined compression test, these two points will serve to define O. Mohr's[17] envelope of rupture for perfectly brittle material (Fig. 6). This envelope can be expressed as Coulomb's equation

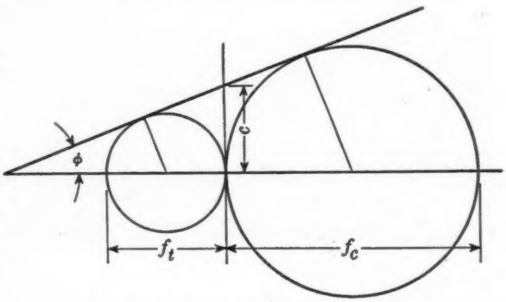


Fig. 6.—Mohr's Envelope of Rupture

$$\tau = c + p \tan \phi \qquad (21)$$

Note.—This paper by E. A. Ripperger and N. Davids was published in February, 1946, *Proceedings*.

[15] Prin. Engr., Ohio River Div. Laboratories, Mariemont, Ohio.

[16] Senior Engr., Ohio River Div. Laboratories, Mariemont, Ohio.

[16a] Received July 1, 1946.

[17] "Abkandlungen aus dem Gebiete der Technischen Mechanik," by O. Mohr, W. Ernst, Berlin, 1914.

in which τ is the unit shearing strength; c is the apparent unit cohesion; p is the unit normal load; and ϕ is the angle of internal friction.

By examination, the following relationship of ϕ and c results in terms of the tensile strength f_t and unconfined compressive strength f_c.

$$c = \frac{f_t}{2 \cos \phi} (1 + \sin \phi) \qquad \text{(22a)}$$

and

$$\phi = \text{arc sin} \frac{f_c - f_t}{f_c + f_t} \qquad \text{(22b)}$$

The application of the ring test for measuring the tensile strength of rock does make use of the assumption (upon which the authors' computations are based) that the tested material is truly elastic. Within the limitations of this assumption a ring specimen prepared and tested as described in this paper will yield the tensile strength of a brittle material by applying an adaptation of Eq. 20:

$$f_t = \sigma_\theta = \frac{P}{\pi r_0} K \qquad \text{(23)}$$

Photographs of nominal rock specimens 3 in. and 6 in. in diameter which have been so tested are shown in Fig. 7. The regularity of the vertical break; the uniformity of results from two specimens, each cut from the ends of a compres-

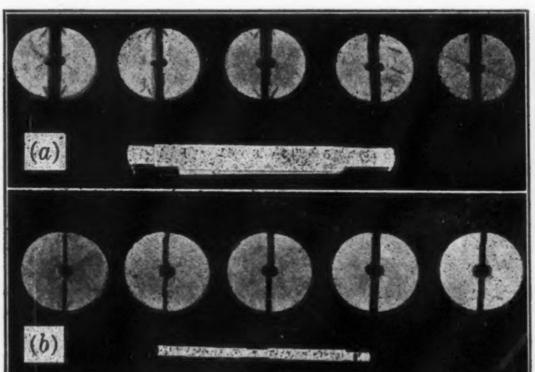


Fig. 7

Fig. 8

sion specimen; the sensitive result, and the plausibility of over-all results—at least make this test useful for comparative purposes.

The compressive strength f_c is measured by a simple, unconfined, compressive test on the rock. A specimen is prepared by cutting a rock core into a length equal to about twice the diameter, then lapping the end surfaces until they are truly perpendicular to the axis of the core. Extreme care in preparation is needed for good results, an outstanding example of which is shown in Fig. 8. The sharpness of the break shown on this photograph suggests another means of measuring the angle of internal friction. The angles of rupture (θ)

according to J. B. Johnson[18] should bear the following relationship to the angle of internal friction (ϕ):

$$\phi = 2\,(\theta - 45°) \qquad (24)$$

Generally the angles found in this manner are from 2° to 5° greater than those found by the measurement of f_t and f_c. In the case of the Sutton sandstone, illustrated in Fig. 8, the angle of fracture varied between 68° and 70°. By Eq. 24 the angle of internal friction is computed to be between 46° and 50°, as against the value of 46° determined by the measurement of f_t and f_c.

TABLE 3.—TENSILE STRENGTH, WITH COMPUTED VALUES OF THE ANGLE OF INTERNAL FRICTION ϕ AND THE COHESION COEFFICIENT c, FOUNDATION ROCK FROM SELECTED DAM SITES

Dam site	Type of foundation rock	Void ratio[a]	POUNDS PER SQUARE INCH			Internal friction ϕ (degrees)
			σ_c	σ_t	c	
Mining City, Ky......	Sandstone	0.258	3,050	762	762	37
Mining City, Ky......	Sandstone	0.135	6,150	1,770	1,665	34
Sutton, W. Va........	Hard gray sandstone	0.077	9,470	1,537	1,907	46
Berlin, Ohio..........	Brown, fine-grained sandstone	0.184	7,280	1,244	1,500	45
Berlin, Ohio..........	Gray, medium-grained, friable sandstone	0.232	5,720	1,116	1,268	42
Center Hill, Tenn.....	Hard, fossiliferous limestone	0.019	8,326	3,540	2,715	23
Wolf Creek, Ky........	Hard, gray limestone	0.013	13,100	5,090	4,082	26
Falmouth, Ky..........	Medium, crystalline, very fossiliferous limestone	0.019	15,200	4,280	4,030	34
Buggs Island, Va......	Gray, granatoid gneiss	....	18,607	5,950	5,271	31
Conemaugh, Pa........	Sandy, medium hard siltstone	0.036	5,920	2,215	1,811	27
Allatoona, Ga.........	Dolomite	0.005	25,400	5,530	5,930	40
Allatoona, Ga.........	Quartzite	0.007	21,300	5,410	5,366	37

[a] Ratio of the volume of voids to the volume of the solids.

Table 3 lists some dam sites at which foundation investigations have yielded 3-in. and 6-in. rock cores that have been tested by the measurement of f_t using the ring test and f_c. The results are encouraging in so far as they are consistent with the classification and the relative appearance of these rocks. In addition there is a degree of correlation between the type of rock, its void ratio, and its physical properties as revealed by this method.

Further means of checking this procedure can be obtained by conducting triaxial tests on cylindrical specimens. Steps in that direction are already being taken by the staff of the Ohio River Division Laboratories. However, the triaxial test requires apparatus and technique far too complicated for normal use; it is anticipated that its greatest value will be in obtaining measurements of stress and strain on the more critical specimens revealed by the ring and compression test results.

The need for a dam in some general locality is not often fixed by the quality of its foundation. The increasing size of dams, the complication of types of concrete sections, and the price competition of earth dams are making it more necessary to take advantage of all there is to know of the physical properties of the materials of construction. It is believed that the ring test for tensile strength determinations will be a very useful means of judging these properties.

[18] "The Materials of Construction," by J. B. Johnson, John Wiley & Sons, Inc., New York, N. Y., 1898 p. 25.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRENGTH OF THIN STEEL COMPRESSION FLANGES

Discussion

BY ROBERT L. LEWIS AND DWIGHT F. GUNDER

ROBERT L. LEWIS,[23] JUN. ASCE., AND DWIGHT F. GUNDER,[24] ASSOC. M. ASCE.[24a]—A basis for the design of structural members fabricated from thin sheets of steel has been presented by Professor Winter. Any design procedure should be kept as simple as is consistent with safe and economical practice. With this thought in mind the writers have examined the proposed development, hoping to find a somewhat simpler approach for the designer. It should be emphasized that it is infinitely more simple to view a problem of this nature in retrospect, and that any contributions which this paper may make will owe their origin to the basic work of Professor Winter.

The author presents experimental evidence that confirms E. E. Sechler's data showing that the constant term in the von Kármán formula, Eq. 2*a*, should be replaced by a variable coefficient C which is a function of $(t/b)\sqrt{E/S}$. This leads to the expression $b_e = C\,t\sqrt{E/S}$ (Eq. 2*b*). Dividing both sides by b, Eq. 2*b* reduces to the dimensionless form,

$$\frac{b_e}{b} = \frac{C\,t}{b}\sqrt{\frac{E}{S}} \qquad (14)$$

As stated by the author, C appears to be a function of $(t/b)\sqrt{E/S}$. Hence Eq. 14 can be expressed as

$$\frac{b_e}{b} = \frac{t}{b}\sqrt{\frac{E}{S}}\,f_1\,\frac{t}{b}\sqrt{\frac{E}{S}} \qquad (15)$$

which merely states that

$$\frac{b_e}{b} = f\left(\frac{t}{b}\sqrt{\frac{E}{S}}\right) \qquad (16)$$

NOTE.—This paper by George Winter was published in February, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1946, by Fred T. Llewellyn, and Jacob Karol.

[23] Prof. and Head, Civ. Eng., Colorado Agri. and Mech. College, Fort Collins, Colo.

[24] Prof. of Graduate Eng., Colorado Agri. and Mech. College, Fort Collins, Colo.

[24a] Received August 19, 1946.

Referring to Fig. 2 in which experimental values of the coefficient C are plotted against the parameter $(t/b)\sqrt{E/S}$, it will be noted that

$$\frac{C\,t}{b}\sqrt{\frac{E}{S}} = \frac{b_e}{t}\sqrt{\frac{S}{E}} \qquad (17a)$$

and

$$\frac{t}{b}\sqrt{\frac{E}{S}} = \frac{b_e}{b} \qquad (17b)$$

The experimental values shown in Fig. 2 were read as carefully as possible from the graph and all values of the coefficient were multiplied by corresponding values of the parameter, thus obtaining the experimental values of b_e/b.

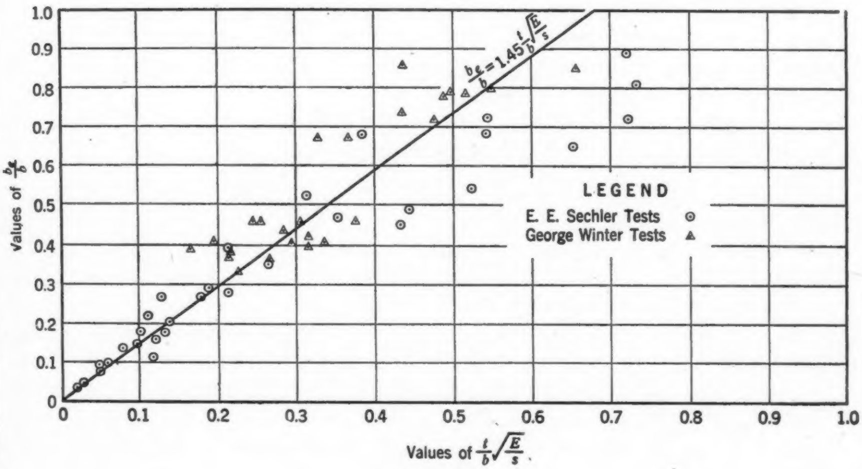


FIG. 17

These values were plotted against the parameter $(t/b)\sqrt{E/S}$ as shown in Fig. 17. As a first approximation to the function $f[(t/b)\sqrt{E/S}]$ the best straight line was fitted to these data by the method of least squares. Its equation is:

$$\frac{b_e}{b} = 1.45\,\frac{t}{b}\sqrt{\frac{E}{S}}; \qquad 0 \leqq \left(\frac{t}{b}\right)\sqrt{\frac{E}{S}} \leqq 0.7 \qquad (18)$$

In order to compare Eq. 6 with Eq. 18 the test values of Fig. 2 were plotted in Fig. 5 and on a graph of Eq. 18. Standard deviations from each curve were computed. The standard deviation from Professor Winter's curve was found to be 0.084 and that of the straight line was 0.113. The difference in standard deviations is caused by the wide scatter of three points of the Sechler data for which the value of b_e/b was approximately equal to 1. Although it is felt that with the limited data available the straight-line approximation is probably as satisfactory as any other, the F-test for goodness of fit does show that Professor Winter's curve is significantly better than Eq. 18. As a refinement to Eq. 18,

a second-degree expression for b_e/b was fitted directly to the given data, as follows:

$$\frac{b_e}{b} = 1.8\frac{t}{b}\sqrt{\frac{E}{S}} - 0.9\left(\frac{t}{b}\sqrt{\frac{E}{S}}\right)^2 \text{; standard deviation} = 0.082 \ldots (19)$$

Eq. 19 corresponds to Eq. 6 of the paper.

Although this curve is a slightly better fit than Eq. 6, it is not significantly so. Noting the apparently wide discrepancy in coefficients in Eq. 19 and Eq. 6 which causes no appreciable change in fit, one sees that the scatter of the points is so large for a b_e/b-ratio of nearly unity that more exact methods are scarcely justified.

In view of the foregoing discussion and the fact that a more careful study of the behavior of b_e/b is needed in the range $0.7 \leqq b_e/b \leqq 1$ it is suggested that, until more data are available, Eq. 18 be used for design purposes. This eliminates the necessity for the design curves given in Fig. 7 and permits direct use of Eq. 18 itself in design problems.

In conclusion the writers would like to call attention to a few mechanical improvements which could be made in the paper:

1. Fig. 7 would perhaps be more useful if the ordinates were values of b_e/b rather than of b_e/t.

2. The number of significant figures is misleading in several places, as, for example, in Eqs. 7 where an approximate value of 25 is used in combination with coefficients such as 1.0906.

3. Averages of the ratios d_1/d_t and d_2/d_t were the criteria used to judge the validity of using the equivalent width as a basis for determining deflections. Actually, it is the standard deviations of the values of d_1/d_t and d_2/d_t about these means which are the more significant measures. In the light of these deviations, the use of b_e for the calculations of deflections is far more superior than is indicated in the paper.

4. Finally, not only in this paper but in all publications it would be extremely helpful if standard deviations were included whenever mean values or fitted curves are used, since these give the reader a much more accurate picture of the reliability of the results.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF UNSYMMETRICAL BEAMS BY THE METHOD OF SEGMENTS

Discussion

By William A. Conwell, Ralph W. Hutchinson, and Thomas P. Revelise

William A. Conwell,[4] M. ASCE.[4a]—The treatment given by Mr. Lifsitz to this important subject has the virtue of maintaining close contact with fundamentals and thus eliminating elaborate tables. Thus, it appeals to the engineer who prefers to visualize structural action while making his calculations rather than to read results from tables and then interpret them. A favorable comparison between the results of the method of segments and the so-called "exact" methods referred to by the author in the "Synopsis" and "Summary" is not dependent upon any approximation inherent in the method. The only element entering the procedure and likely to affect the accuracy is the degree to which the shape of the beam upon which tables or charts (such as those of Figs. 8 and 9) are based conforms to that of the beam being analyzed.

Despite the fact that the material of the paper is not unduly complicated, it is not easily read. The reason for this difficulty may be traced to two items—organization and sacrifice of clarity and fullness for brevity.

Unlike the "Typical Example" which is superlatively organized, the preceding theory does not proceed logically to a solution. If the theory were to parallel the "Typical Example" there would first be a treatment of "Stiffness and Carry-Over Factors" and then, under "Fixed-End Moments," a sharp division of the steps of the procedure as follows:

1. Bending moments in segmental spans AO and OB resulting from load P;
2. Bending moments in beam AB resulting from a vertical load at point O; and
3. Fixed-end moments in beam AB resulting from load P.

A clear statement of the author's intention at the beginning of each division would aid the reader immeasurably.

An example of brevity at the expense of clarity is evident in the second paragraph under "Fixed-End Moments." The first sentence—"Assume a

Note.—This paper by Sol Lifsitz was published in March, 1946, *Proceedings*.

[4] Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

[4a] Received May 2, 1946.

vertical settlement, y_o, at point O"—immediately raises a question. "Settlement" is usually associated with the relaxation of action or removal of a support of a loaded beam. In the instance under consideration, removal of the temporary support at point O would cause no settlement because the beam is not loaded. If, then, it is desired to produce a deflection at point O, it must be done by positive action—that is, by placing loads on the beam. Only one condition of loading—namely, a vertical load at point O—fits the requirements of the problem, however. It is suggested, therefore, that this step in the process of determining fixed-end moments for span AB be considered merely the computation of the bending moments at points A and B resulting from the application of a vertical load at point O. Thus, y_o is considered a means rather than an end and is relegated to the background where it really belongs.

In omitting the derivation of Eq. 2 brevity is achieved at the expense of fullness and the introduction of rich background material. Although this relation has appeared elsewhere,[5] it would seem more in keeping with the author's practice of dealing in fundamentals if a more detailed reference or a demonstration were included. The relation goes much deeper than the "principles" of moment distribution which really furnish only the definitions of the terms. The full implication of the equation is apparent only upon return to the principle that the amount of work done by forces acting upon a body is independent of the order of application of the forces. A demonstration similar to the following would seem apropos:

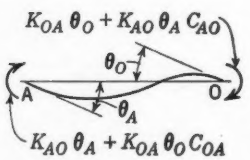


FIG. 11

If a moment $K_{AO}\,\theta_A$ is applied at end A of a beam AO while point O is restrained against rotation, it will produce a rotation θ_A at end A and induce a bending moment, $K_{AO}\,\theta_A\,C_{AO}$, at point O. A similar application of a moment $K_{OA}\,\theta_O$ at point O while end A is restrained will produce a rotation of θ_O at point O and induce a bending moment, $K_{OA}\,\theta_O\,C_{OA}$, at end A. The beam AO under the simultaneous action of these moments is shown in Fig. 11. The work (W) by the application of end moments in the order $K_{AO}\,\theta_A$, $K_{OA}\,\theta_O$ is

$$W = \tfrac{1}{2}\,K_{AO}\,\theta^2_A + \tfrac{1}{2}\,K_{OA}\,\theta^2_O + K_{AO}\,\theta_A\,C_{AO}\,\theta_O \quad \text{(20a)}$$

Similarly, application in the order $K_{OA}\,\theta_O$, $K_{AO}\,\theta_A$ gives for the amount of work,

$$W = \tfrac{1}{2}\,K_{OA}\,\theta^2_O + \tfrac{1}{2}\,K_{AO}\,\theta^2_A + K_{OA}\,\theta_O\,C_{OA}\,\theta_A \quad \text{(20b)}$$

Equating these expressions gives: $\tfrac{1}{2}\,K_{AO}\,\theta^2_A + \tfrac{1}{2}\,K_{OA}\,\theta^2_O + K_{AO}\,\theta_A\,C_{AO}\,\theta_O = \tfrac{1}{2}\,K_{OA}\,\theta^2_O + \tfrac{1}{2}\,K_{AO}\,\theta^2_A + K_{OA}\,\theta_O\,C_{OA}\,\theta_A$; and

$$(K_{AO}\,\theta_A\,C_{AO})\,\theta_O = (K_{OA}\,\theta_O\,C_{OA})\,\theta_A \quad \text{(21)}$$

In general terms, Eq. 21 states that the bending moment at point O, induced by a rotation θ_A, multiplied by the rotation θ_O, equals the bending moment at end A, induced by a rotation θ_O, multiplied by the rotation θ_A. In particular,

[5] "Design Constants for Beams with Nonsymmetrical Straight Haunches," by August L. Ahlf, *Transactions*, Vol. 110, 1945, p. 1019.

if $\theta_A = \theta_O = 1$,

$$K_{AO}\, C_{AO} = K_{OA}\, C_{OA} \qquad (22)$$

and a statement of this relation becomes simply: The bending moment at point O induced by a unit rotation at end A equals the bending moment at end A induced by a unit rotation at point O. The principles involved are evidently those of the Maxwell reciprocal theorem.

Clarity again appears to be sacrificed in the twelfth paragraph under "Fixed-End Moments" in the sentence:

> "Since the force required to restore point O to its original position is evidently equal to $F_{p,OA}$, it is clear that an equal but opposite force is acting downward on the beam at point O after removal of the support."

It is suggested that this step, after computation of the reaction $F_{p,OA}$, be considered as follows:

1. Replace the reaction, $F_{p,OA}$, with an equal, upward-acting load $F_{p,OA}$. Of course, this action will leave the state of stress and deflection in the beam precisely as it was with the reaction in place; but it may now be treated as a single span AB.
2. By the method outlined in the discussion of the second paragraph under "Fixed-End Moments," compute the end moments resulting from a vertical downward load at point O equal to $F_{p,OA}$ and acting alone on beam AB.
3. Superimpose the loading and bending conditions of the second paragraph on those of the first paragraph. The equal and opposite loads at point O cancel each other and the bending moments are those resulting from P only.

Although Figs. 8 and 9 could have been omitted, their inclusion is commendable in that they round out the paper and make it a fairly complete tool. Values more accurate than those that can be read from the diagrams may often be desired but they are readily available in standard tables, as the author states. A description of the shape of beam upon which Figs. 8 and 9 were based would be of value. The application of tables for straight haunches[5] to the beam of the "Typical Example" yields the values $K_{AB} = 0.294\,E$, $C_{AB} = 0.541$, $K_{BA} = 0.159\,E$, and $C_{BA} = 0.997$. When these values are compared with those of the "Typical Example" ($K_{AB} = 0.168\,E$, $C_{AB} = 0.583$, $K_{BA} = 0.105\,E$, and $C_{BA} = 0.933$), some idea is obtained of the extreme variations in these characteristics resulting from differences in shape only.

In view of the foregoing it may be concluded that, despite reading made difficult by easily-removable obstacles, this paper has a more-than-ordinary value and that the presentation of the "Typical Example" is far superior to that of the remainder of the paper.

RALPH W. HUTCHINSON,[6] Assoc. M. ASCE.[6a]—The method of segments is similar to a method developed by engineers of the writer's organization for obtaining the properties of beams with intermediate hinges from those of the solid beam, or the same beam without the hinge. The properties of the solid

[6] Associate Bridge Engr., State of California, Sacramento, Calif.

[6a] Received July 1, 1946.

beam can usually be obtained from published charts whereas available data on beams with hinges are meager.

As in the method of segments the elastic area is first obtained, together with the moment of inertia about its centroid and the location of the centroid. Because of the similarity between a beam with an intermediate hinge and one with a hinge at one end, the formulas used were based on the properties of a beam hinged at one end. By this method a formula for the elastic area A may be expressed in terms of properties of the beam without first solving for the location of the centroid. For comparison with the formula used by the author, this formula may be written in terms of a beam fixed at both ends by including the factor $(1 - C_{AB} \times C_{BA})$:

$$\frac{A}{(1 - C_{AB} \times C_{BA})} = \frac{1 - C_{AB}}{K_{BA}} + \frac{1 - C_{BA}}{K_{AB}} \qquad (23)$$

The properties of the hinged beam are computed by first finding the moment of inertia of the elastic area about the hinge. In the following formulas, the properties of the hinged beam are indicated by adding the letter H to the letter symbols used for these same properties in the solid beam: $c = \bar{x} - a$; $I_H = I + A\,c^2$; $K_{ABH} = \frac{a^2}{I_H}$; $K_{BAH} = \frac{b^2}{I_H}$; $C_{ABH} = \frac{b}{a}$; $C_{BAH} = \frac{a}{b}$; and $d = \frac{I_H}{C\,A}$.

Applying the foregoing equations to the solid beam in the Lifsitz paper: $A = 58.8$; $I = 2{,}612$; $\bar{x} = 19.85$; $a = 7$ ft; $b = 28$ ft; $c = 19.85 - 7.00 = 12.85$ ft; $I_H = 2{,}612 + 58.8 \times 12.85^2 = 12{,}332$; $K_{ABH} = \frac{7 \times 7}{12{,}332} = 0.00397$; $K_{BAH} = \frac{28 \times 28}{12{,}332} = 0.0636$; $C_{ABH} = \frac{28}{7} = 4$; $C_{BAH} = \frac{7}{28} = 0.25$; and $d = \frac{12{,}320}{12.85 \times 58.8} = 16.3$ ft. From the tables in Mr. Lifsitz' paper $(\text{FEM})_A = 8.82$; and $(\text{FEM})_B = 1.69$. For graphical solution the analyst can plot a simple beam moment diagram and draw a closing line through the fixed-end moments for the solid beam. The closing line for the hinged condition is drawn through the simple beam moment curve at the hinge and through the fixed-end moment line at the conjugate point.

After the properties of the beam have been obtained, fixed-end moments may be obtained by either graphical or analytical means. An illustrative example of the graphical method is given in which a hinge is inserted at the fifth-point in the beam used by the author to illustrate the method of segments. The fixed-end moments are computed for a load of $P = 1$ at 12 ft from end A (Fig. 12).

Time often does not permit an exact determination of the properties of beams of unusual shape, and every engineer has had to rely on approximations extrapolated from working tables. Even though these approximations are usually close enough for the purpose there is a real need for a quick method of obtaining results with a precision equal to data picked from charts for regular beams. The method of segments gives promise of meeting this need, but the work required for the computation of fixed-end moments is disappointing. Some engineers may find it economical to use the method of segments to de-

termine the elastic properties of the beam and then compute fixed-end moments by a more familiar method.

Before deciding for or against the adaptation of the method of segments as a working tool, one should compare the work involved in a complete solution with

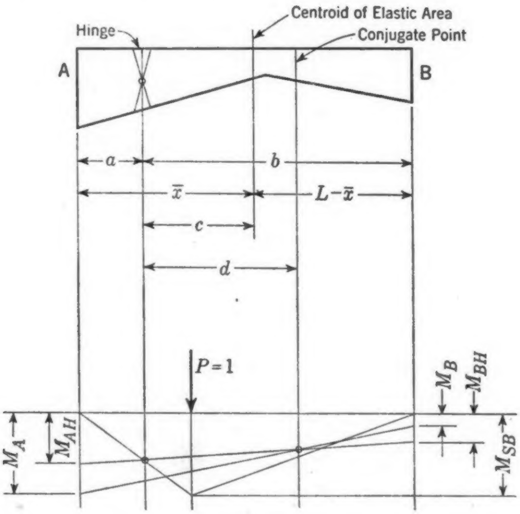


Fig. 12

the work required by a method of "multiplying factors." Such a method was developed by Carl Wagner, Jr., Jun. ASCE, from contributions by George E. Large[7] and A. W. Earl,[8] Members, ASCE.

Thomas P. Revelise,[9] Esq.[9a]—The technique of computing design constants for unsymmetrical beams has been advanced perceptibly by the publication of this paper. However, the writer does not consider the method as general in scope as the author implies (see "Summary"):

> "By readily available curves similar to those in Fig. 9, together with a few simple equations, influence-line data for fixed-end moments for any unsymmetrical beam may be quickly tabulated."

Curves cannot be constructed either for use alone or in conjunction with supplementary methods for computing beam design constants if the shape or cross section of the member is sufficiently discontinuous. Several important types occur in this category. A familiar example is the continuous plate girder, with multiple cover plates of varying lengths, placed unsymmetrically over supports and elsewhere in the spans. In this case, the moments of inertia at short successive sections are both variable and discontinuous, and there is no

[7] *Transactions*, ASCE, Vol. 96, 1932, p. 101.

[8] *Ibid.*, p. 112.

[9] Highway Bridge Engr., U. S. Public Roads Administration, Atlanta, Ga.

[9a] Received July 8, 1946.

short cut to a rational solution. The problem is also encountered occasionally in the design of reinforced concrete beams whose cross section at successive points is discontinuous.

The few exceptions do not detract from the merit of Mr. Lifsitz's method and other short-cut methods, which are applicable to most cases encountered in routine structural practice; but it is well to remember that these methods are necessarily based on certain assumptions of geometric continuity, which, if not fulfilled, make recourse to a so-called "exact" analysis the only remaining alternative.

In view of the considerable interest in this subject, evidenced by various published papers, the writer would like to say a word about its most neglected phase—the exact procedure. Most writers seem agreed that an exact procedure is too involved and unwieldy for practical use. The reader is invited to accept this premise without question, since, as a rule, no comparative demonstration is given. Although most textbooks on structural analysis present the theoretical fundamentals, few writers give consideration to a well-coordinated exact procedure arranged to yield, with a minimum of computation, the answers in which the average designer is primarily interested—namely, the stiffness and carry-over constants at both ends of a given beam and the (FEM)-coefficients at both ends for a unit load at the tenth points (or the equivalent, unit influence lines for (FEM) at both ends).

If properly organized, an exact procedure for the determination of beam design constants is much less formidable than is generally appreciated, and its applicability to all cases makes it a valuable tool for anyone engaged in this phase of structural design. Practically any variant of the elastic theory may be used in deriving the necessary expressions. Writers of today show a preference for the method of column analogy. Other variants, such as moment areas, least work, or slope deflection, can be used with fully comparable results.

Several years ago, the writer arranged a tabular form for the analysis of unusual and borderline cases, which is offered as an example of what can be prepared along this line.

The basic expressions were obtained by developing, algebraically, the fundamental elastic equation—

$$d\phi = \int \frac{M\,ds}{E\,I} \qquad (24)$$

—and using physical summations instead of integrals. If approached in this manner, the most advanced mathematical operation is found to be the solution (for this problem) of two simple simultaneous equations. The resultant expressions were then separated into component parts and rearranged in orderly fashion in a tabular form which is nonmathematical in character and can be executed without reference to the theoretical derivations.

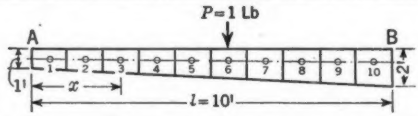
The member to be analyzed is first drawn to a convenient scale and divided into ten sections of equal horizontal length as shown in Table 3. The depth, d, at the center of each section is scaled, and values of $I = \frac{b\,d^3}{12}$ are computed

TABLE 3.—COMPUTATION OF CONSTANTS FOR BEAMS WITH VARIABLE MOMENT OF INERTIA (UNIT LOADS SUCCESSIVELY AT POINTS x; WIDTH OF BEAM, 1 FT)

$Z_A = AC - B^2 = 1.42$

$K_A = \frac{C}{Z_A l} = 0.058$

$C_A = 1 - \frac{B}{C} = -0.83$



$M_f = f P l$

$K E$ = absolute stiffness[a]

$K_B = \frac{E}{Z_A l} = 0.161$

$C_B = 1 - \frac{D}{E} = -0.30$

x (point No.) (1)	I (2)	$\frac{x}{l}$ (3)	$\frac{0.1}{I}$ (4)	Col. 3 times Col. 4 (5)	Col. 3 times Col. 5 (6)	Σ Col. 4 minus Col. 4_x (7)	Σ Col. 7 minus Col. 7_{x-1} (8)	Σ Col. 5 minus Col. 5_x (9)	Σ Col. 9 minus Col. 9_{x-1} (10)	Col. 10 times B (11)	Col. 8 times C (12)
1	0.097	0.05	1.03	0.05	0	3.44	12.76	1.45	7.38	11.07	10.46
2	0.127	0.15	0.79	0.12	0.02	2.65	9.32	1.33	5.93	8.90	7.64
3	0.163	0.25	0.61	0.15	0.04	2.04	6.67	1.18	4.60	6.90	5.47
4	0.205	0.35	0.49	0.17	0 06	1.55	4.63	1.01	3.42	5.13	3.80
5	0.254	0.45	0.39	0.18	0.08	1.16	3.08	0.83	2.41	3.62	2.53
6	0.311	0.55	0.32	0.18	0.10	0.84	1.92	0.65	1.58	2.37	1.57
7	0.375	0.65	0.27	0.18	0.12	0.57	1.08	0.47	0.93	1.40	0.89
8	0.447	0.75	0.22	0.16	0.12	0.35	0.51	0.31	0.46	0.69	0.42
9	0.528	0.85	0.19	0.16	0.14	0.16	0.16	0.15	0.15	0.23	0.13
10	0.619	0.95	0.16	0.15	0.14	0	0	0	0	0	0
Σ	....	....	4.47	1.50	0.82	12.76	....	7.38	....	....	....
Design constants		....	A	B	C	....	....	....	....	....	....

TABLE 3.—(*Continued*)

x (point No.) (1)	Col. 11 minus Col. 12 (13)	Col. 13 times $\frac{1}{10 Z_A}$ (14)	Col. 14 times A (15)	Col. 15 plus Col. 8/10 (16)	Col. 16 times $1/B$ (17)	1 minus Col. 3 (18)	Col. 17 minus Col. 18 (19)	Col. 14 minus Col. 19 (20)	Col. 18 times Col. 4 (21)	Col. 18 times Col. 21 (22)
1	0.61	0.043	0.192	1.469	0.980	0.95	+0.030	0.013	0.98	0.93
2	1.26	0.089	0.398	1.330	0.887	0.85	+0.037	0.052	0.67	0.57
3	1.43	0.01	0.451	1.118	0.746	0.75	−0.004	0.105	0.46	0.35
4	1.33	0.094	0.420	0.883	0.589	0.65	−0.061	0.155	0.32	0.21
5	1.09	0.077	0.344	0.652	0.435	0.55	−0.115	0.192	0.21	0.12
6	0.80	0.056	0.250	0.442	0.295	0.45	−0.155	0.211	0.14	0.06
7	0.51	0.036	0.161	0.269	0.179	0.35	−0.171	0.207	0.09	0.03
8	0.27	0.019	0.085	0.136	0.091	0.25	−0.159	0.178	0.06	0.02
9	0.10	0.007	0.031	0.047	0.031	0.15	−0.119	0.126	0.03	0
10	0	0	0	0	0	0.05	−0.050	0.050	0.01	0
Σ	....	....	....	....	....	....	....	....	2.97	2.29
Design constants		f_A	....	....	....	....	....	f_B	D	E

[a] K × modulus of elasticity equals absolute stiffness.

and entered in Col. 2, Table 3. Successive operations are indicated under each column heading; and, upon completion of the form, all necessary design constants, including fixed-end moments at both ends for unit concentrations at the tenth points are obtained. The meaning of the headings for Cols. 7 to 10 may be further clarified by reference to point $x = 3$, as follows:

$$\text{Col. } 7 = \Sigma \text{ Col. } 4 - \text{Col. } 4_x = 4.47 - (1.03 + 0.79 + 0.61) = 2.04$$

$$\text{Col. } 8 = \Sigma \text{ Col. } 7 - \text{Col. } 7_{x-1} = 12.76 - (3.44 + 2.65) = 6.67$$

$$\text{Col. } 9 = \Sigma \text{ Col. } 5 - \text{Col. } 5_x = 1.50 - (0.05 + 0.12 + 0.15) = 1.18$$

$$\text{Col. } 10 = \Sigma \text{ Col. } 9 - \text{Col. } 9_{x-1} = 7.38 - (1.45 + 1.33) = 4.60$$

The example used to illustrate the procedure was chosen to afford a comparison between computed values and those obtained from available curves. The procedure remains the same, regardless of what form the dissymmetry or discontinuity takes.

Forms of the type of Table 3 can be derived independently by other methods, or the form illustrated can be altered or modified to suit the needs or preferences of the user.

Corrections for *Transactions:* In March, 1946, *Proceedings*, on page 324, line 6, change "AB" to "OB" and "fixed-end moments" to "final end moments"; "F_o" to "F_y" in two places in Fig. 2 and in the unnumbered equation preceding Eqs. 20; "m''_{AO}" to "m''_{OA}" in Fig. 3(*b*); "m''_{OA}" to "m''_{AO}" in Fig. 3(*c*); "F_{pA}" to "$F_{p,OA}$" in Fig. 4; "F_{pB}" to "$F_{p,OB}$" in Fig. 5; "K" to "κ" in Eqs. 9 and 12; on page 327, line 11, "I" to "I'"; "h_h" to "rh_o" in Fig. 6 ($h_h = h_o = rh_o$) and "$\bar{X}$," "$\bar{X}_a$," and "$\bar{X}_b$" to "$\bar{x}$," "$\bar{x}_a$," and "$\bar{x}_b$"; "h_r" to "r" in Fig. 9; on page 333, the values corresponding to $\kappa = 0.4$ in Cols. 12 and 14, Table 1, from "3.05" to "2.98" and "8.82" to "8.75," respectively; on page 334, line 32, "$E^x I$" to "$E\ I_x$"; and, on page 335, lines 9 and 10, change "right end" and "left end" to "hinged end (see Fig. 3)" and "fixed end (see Fig. 3)," respectively. Delete the first parenthesis in the second term of the numerator of Eq. 19*d*.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TORSION IN STEEL SPANDREL GIRDERS

Discussion

By Robert V. Hauer

Robert V. Hauer,[10] Esq.[10a]—The method of computing torsional stresses in spandrels, as described by the author, is sufficiently accurate, if applied to a concrete construction with the necessary modifications; but it is not satisfactory in the case of steel beams.

Even if the spandrel is assumed to be connected to the columns in such a way that the end cross sections are free to warp, there must be one section in the central part of the spandrel which remains plane and which is therefore the origin of a certain stress irregularity. In steel beams this causes a considerable increase of the torsional stiffness, changes the distribution of the shear stresses, and gives rise to normal stresses in the middle of the girder. The latter are of much more interest than the shear stresses, because they are additional to the bending stresses that generally govern the design of the spandrel.

Checking Example 3, by the use of the approximate method[11] of S. Timoshenko, shows that the torsional stiffness of the spandrel is 2.11 times as great as that computed by the author and that the additional normal stresses in the flanges amount to 2,780 lb per sq in.—a quantity which can hardly be considered negligible. The shear stress in the web due to torsion alone is found to be 43% higher than the value given by the author.

In principle, the behavior of a rectangular spandrel as used in concrete is the same. However, the difference is that, in a rectangular beam, a stress irregularity in a certain section practically disappears within a very short distance from this section and therefore does not materially affect the member as a whole, whereas an I-beam, because of its particular shape, emphasizes such irregularities. In the quarter points of the spandrel investigated in Example 3—that is, at a distance from the center equal to about three and one-half times the depth of the beam—the additional normal stress in the flanges due to torsion is still 35% of its maximum value. It might be stated

Note.—This paper by J. E. Lothers was published in March, 1946, *Proceedings*. Discussion of this paper has appeared in *Proceedings*, as follows: June, 1946, by John E. Goldberg, and I. Oesterblom.

[10] Structural Engr., Albert Kahn, Inc., Detroit, Mich.

[10a] Received May 22, 1946.

[11] *Zeitschrift Mathematik Physik*, Vol. 58, 1910, p. 361.

that an I-beam is somewhat reluctant to follow the well-known principle of de Saint-Venant.

Since the torsional stiffness of any part of an I-spandrel depends on the degree of restraint against warping of its end sections, this stiffness cannot generally be determined in advance, but is governed by the final distribution of the torsional moments in the entire spandrel. Therefore, both the method of moment distribution and the slope-deflection method fail in problems like those discussed by the author, except in the cases where only one floor beam or two symmetrical floor beams frame into the spandrel. In all other cases only the classical method of redundant moments offers a solution.

Correction for *Transactions:* On page 314, fourth line from the bottom of the page change "* * * $M_{AB} = \frac{51}{51 + 733}$ * * *", to "* * * $M_{AB} = \frac{51}{51 + 773}$ * * *."